

ULEAC/ULSEB Special Paper Pure Mathematics Mark Schemes – June 1992 to June 1995

Question Number	Scheme	Marks
<p>1. (a)</p> <p>(b)</p>	$0.\dot{4} = 0.4 + 0.4(0.1) + 0.4(0.1)^2 + \dots$ <p>Geometric series $a = 0.4, r = 0.1$</p> $S_{\infty} = \frac{0.4}{1-0.1} = \frac{0.4}{0.9} = \frac{4}{9}$ <p>($\therefore p = 4, q = 9$)</p> $T_n = \left(\frac{4}{9} - \frac{4}{9 \times 10^n} \right) 10^n$ $S_n = \frac{4}{9} (10 + 10^2 + \dots + 10^n) - \frac{4}{9} n$ $= \frac{4}{9} \left(\frac{10(1-10^n)}{1-10} \right) - \frac{4}{9} n$ $= \frac{4}{9} \left(\frac{10}{9} (10^n - 1) - n \right) \quad \left[= \frac{4}{81} (10^{n+1} - 10 - 9n) \right]$	<p>M1</p> <p>A1</p> <p>M1 A1</p> <p align="right">(4)</p> <p>M1 A1 A1</p> <p>M1 A1 A1</p> <p>M1 A1</p> <p>A1</p> <p align="right">(9)</p> <p align="right">(13 marks)</p>
<p>Alt (b)</p>	$T_n = 4(1 + 10 + 100 + \dots + 10^n) = \frac{4}{9} (10^{n+1} - 1)$ $S_n = \frac{4}{9} \sum_{m=0}^n 10^{m+1} - \frac{4}{9} \sum_{m=0}^n 1$ $= \frac{4}{81} (10^{n+1} - 1) - \frac{4}{9} (n+1) \quad \left[= \frac{4}{81} (10^{n+1} - 10 - 9n) \right]$	<p>M1 A1 A1</p> <p>M1 A1 A1</p> <p>M1 A1 A1</p> <p align="right">(9)</p>

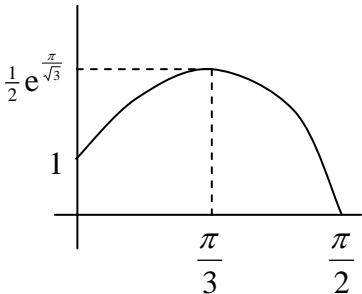
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2.	<p>(a) $b^2 - 4ac = (4(k + 3))^2 - 4(4)(5k + 8)$ $= 16(k^2 + k + 1)$ $= 16\left(\left(k + \frac{1}{2}\right)^2 + \frac{3}{4}\right)$</p> <p>which is positive, $\therefore \alpha, \beta$ real and different</p>	M1 A1 M1 A1 (4)
	<p>(b) $(\alpha - \beta) = (\alpha + \beta)^2 - 4\alpha\beta$ $= (k + 3)^2 - 4\left(\frac{5k + 8}{4}\right)$ $= k^2 + k + 1$ $= \left(k + \frac{1}{2}\right)^2 + \frac{3}{4}$</p>	M1 A1 M1
	<p>$\therefore \alpha - \beta$ least when $k = -\frac{1}{2}$, $\therefore \alpha - \beta = \frac{\sqrt{3}}{2}$</p>	A1 (4)
	<p>(c) $(\alpha + \beta) = k + 3$ $\alpha\beta = \frac{5k + 8}{4}$</p> <p>If $\alpha = 2\beta$ then equations become</p> $\left. \begin{array}{l} 3\beta = k + 3 \\ 2\beta^2 = \frac{5k + 8}{4} \end{array} \right\}$	M1 A1
	<p>Solving gives $k = 0$ and $\beta = 1$ or $k = -\frac{3}{8}$ and $\beta = \frac{7}{8}$</p>	M1 A1 A1 (5) (13 marks)

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<p>3. (a)</p> <p>(b)</p>	$y = \lambda x(x-1)(x-2) = \lambda x^3 - 3\lambda x^2 + 2\lambda x$ $\frac{dy}{dx} = 3\lambda x^2 - 6\lambda x + 2\lambda$ <p>when $x = 1$, $\frac{dy}{dx} = -\lambda$; when $x = 2$, $\frac{dy}{dx} = 2\lambda$</p> <p>For these to be perpendicular we need</p> $(-\lambda)(2\lambda) = -1$ $\therefore \lambda = \pm \frac{1}{\sqrt{2}}$ $\int_{-m}^{(m+2)} y \, dx = \lambda \left[\frac{x^4}{4} - x^3 + x^2 \right]_{-m}^{m+2}$ $= \lambda \left[\left(\frac{(m+2)^4}{4} - (m+2)^3 + (m+2)^2 \right) - \left(\frac{(-m)^4}{4} - (-m)^3 + (-m)^2 \right) \right]$ $= \lambda \left[\frac{1}{4} (m^4 + 8m^3 + 24m^2 + 32m + 16) - (m^3 + 6m^2 + 12m + 8) + (m^2 + 4m + 4) - \left(\frac{1}{4} m^4 + m^3 + m^2 \right) \right]$ $= 0 \quad (*)$	<p>M1 A1</p> <p>M1 A1 ft</p> <p>M1</p> <p>A1 (6)</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 A1</p> <p>A1 (7)</p> <p>(13 marks)</p>

Question Number	Scheme	Marks
4. (a)	$x^2 - x + 1 = \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$ $x - \frac{1}{2} = \frac{\sqrt{3}}{2} \tan \theta, \quad \therefore \frac{dx}{d\theta} = \frac{\sqrt{3}}{2} \sec^2 \theta$ $\int \frac{4}{3(\tan^2 \theta + 1)} \frac{\sqrt{3}}{2} \sec^2 \theta d\theta = \frac{2}{\sqrt{3}} \theta + k = \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) + k$ $\therefore \lambda = \frac{2\sqrt{3}}{3}$	M1 M1 A1 A1 (4)
(b)	$\frac{1}{x^3 + 1} \equiv \frac{A}{x+1} + \frac{Bx+C}{x^2 - x + 1}$ $\equiv \frac{\frac{1}{3}}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2 - x + 1}$ $\int \frac{1}{x^3 + 1} dx = \frac{1}{3} \ln x+1 + \int \frac{2-x}{3(x^2 - x + 1)} dx$ $= \frac{1}{3} \ln x+1 + \int \frac{4-2x}{6(x^2 - x + 1)} dx$ $= \frac{1}{3} \ln x+1 + \frac{1}{6} \int \frac{1-2x}{x^2 - x + 1} dx + \frac{1}{6} \int \frac{3}{x^2 - x + 1} dx$ $= \frac{1}{3} \ln x+1 + \frac{1}{6} \ln x^2 - x + 1 + \frac{1}{2} \int \frac{1}{x^2 - x + 1} dx$ $= \frac{1}{3} \ln x+1 + \frac{1}{6} \ln x^2 - x + 1 + \frac{1}{2} \left(\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) \right) + c$ $= \frac{1}{3} \ln x+1 + \frac{1}{6} \ln x^2 - x + 1 + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}} \right) + c$	M1 A1 A1 A1 M1 M1 A1 (7)
(c)	$\int_1^2 \frac{1}{x^3 + 1} dx = \left(\frac{1}{3} \ln 3 - \frac{1}{6} \ln 3 + \frac{1}{\sqrt{3}} \tan^{-1} \sqrt{3} \right) - \left(\frac{1}{3} \ln 2 - \frac{1}{6} \ln 1 + \frac{1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} \right)$ $= \frac{1}{6} \left(2 \ln 3 - \ln 3 - 2 \ln 2 + \ln 1 \right) + \frac{\sqrt{3}}{3} \left(\frac{\pi}{3} - \frac{\pi}{6} \right)$ $= \frac{1}{6} \ln \frac{3}{4} + \frac{\pi\sqrt{3}}{18} (*)$	M1 A1 (2)
(13 marks)		

Question Number	Scheme	Marks
5.	<p>(a) $y = e^{x\sqrt{3}} \cos x$</p> $\frac{dy}{dx} = -e^{x\sqrt{3}} \sin x + \sqrt{3} e^{x\sqrt{3}} \cos x$ $= 2 e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{6} \right) \quad (*)$ $\frac{d^2y}{dx^2} = 4 e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{3} \right) \quad (*)$ <p>(b) TP when $2e^{x\sqrt{3}} \cos + \left(x + \frac{\pi}{6} \right) = 0 \Rightarrow x = \frac{\pi}{3}, y = \frac{1}{2} e^{\frac{\pi}{\sqrt{3}}}, y'' > 0$</p>  <p>(c) Area = $\int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \, dx = \left[e^{x\sqrt{3}} \sin x \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sqrt{3} e^{x\sqrt{3}} \sin x \, dx$</p> $= e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3} \left(\left[-e^{x\sqrt{3}} \cos x \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \sqrt{3} e^{x\sqrt{3}} \cos x \, dx \right)$ $= e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3} - 3 \int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \, dx$ $\therefore 4 \int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \, dx = e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3}$ $\therefore \int_0^{\frac{\pi}{2}} e^{x\sqrt{3}} \cos x \, dx = \frac{1}{4} \left(e^{\frac{\pi\sqrt{3}}{2}} - \sqrt{3} \right) \quad (*)$	<p>M1</p> <p>M1 A1</p> <p>B1 (4)</p> <p>M1 A1</p> <p>G1 (3)</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (6)</p> <p>(13 marks)</p>

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6. (a)	$\overrightarrow{AQ} = -\mathbf{a} + (1 - \lambda)\mathbf{a} + \lambda\mathbf{b} = \lambda(\mathbf{b} - \mathbf{a})$ which is parallel to \overrightarrow{AB} , $\therefore Q$ lies on \overrightarrow{AB}	B1 (1)
6. (b)	$\mathbf{p} \cdot (\mathbf{b} - \mathbf{a}) = 0 \Rightarrow ((1 - \mu)\mathbf{a} + \mu\mathbf{b}) \cdot (\mathbf{b} - \mathbf{a}) = 0$ $\therefore \mathbf{a} \cdot (\mathbf{b} - \mathbf{a}) - \mu\mathbf{a} \cdot (\mathbf{b} - \mathbf{a}) + \mu\mathbf{b} \cdot (\mathbf{b} - \mathbf{a}) = 0$ $\mathbf{a} \cdot (\mathbf{b} - \mathbf{a}) - \mu(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{a}) + \mu\mathbf{b} \cdot (\mathbf{b} - \mathbf{a}) = 0$ $\mu = \frac{-\mathbf{a} \cdot (\mathbf{b} - \mathbf{a})}{ \mathbf{b} - \mathbf{a} ^2} \quad (*)$	M1 A1 (2)
6. (c)	$\mu = \frac{-\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \cdot \left(\begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right)}{1 + 4 + 1} = \frac{-\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}{6}$ $= -\frac{7}{6}$	M1 A1
6. (d)	$\therefore \mathbf{p} = \frac{13}{6} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} - \frac{7}{6} \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} = \frac{1}{6}(-\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$	M1 A1 (4)
6. (d)	$\overrightarrow{OLAB} = \begin{pmatrix} 1 + \lambda \\ 2 + 2\lambda \\ 2 + \lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}$ $\therefore \frac{7 + 6\lambda}{\sqrt{(1 + \lambda)^2 + (2 + 2\lambda)^2 + (2 + \lambda)^2} \times \sqrt{6}} = \frac{1}{\sqrt{2}}$ $\therefore 7 + 6\lambda = \sqrt{6\lambda^2 + 14\lambda + 9} \times \sqrt{3}$ $\therefore 18\lambda^2 + 42\lambda + 22 = 0$ $\lambda = \frac{-42 \pm \sqrt{1764 - 4 \times 18 \times 22}}{2 \times 18} = \frac{-42 \pm \sqrt{180}}{36}$ $= \frac{-7 \pm \sqrt{5}}{6}$	M1 A1 A1 M1 A1 (6)
		(13 marks)

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7. – 23.	In production – to be circulated when finalised...	

