<u>Exercises in Graph Theory</u> **(9-2-1999, T.W.Leung)**

1. In a party of n persons, $n \geq 3$, at least one person does not shake hand with everybody, determine the maximum number of persons in the party that shake hand with everybody.

2. Given a party of n persons. It is known that from every subgroup of 4 persons, there exists one who knows the other three persons. Determine the minimum number of persons in the group who know everybody.

3. In a conference there are 34 teams, each team consists of a leader and a number. The participants shade hands with others, except the leader of a team does not shake hand with his member. After many hand-shakings the leader of team A asks everybody how many hands they have shaken, and receives distinct answers, how many hand the member of team A has shaken?

4. A contest consists of the first round and the final round, with altogether 28 questions. It is known that each participant solves exactly 7 questions, and every pair of questions is solved by exactly two contestants. Show that there exists a contestant, who either does not solve any question in the first round, or solves at least 4 questions in the first round.

5. There are n points ($n \geq 3$) in the plane and any pair of points is of distance at least 1, show that there are at most $3n-6$ pairs of points whose distances are exactly 1.

6. There are 7 boys and 7 girls in a party, after that they record the number of times they dance : 3, 3, 3, 3, 3, 5, 6, 6, 6, 6, 6, 6, 6, 6. Show that someone has made a mistake.

7. There are n persons sit around a table ($n \geq 6$), show that one can re-arrange the table so that the two persons besides every person are different from the first siting.

8. There are n points on the circle, label them with 1, 2, …, n, so that any two adjacent points differ by at most 2. Can it be done? Is it unique if it can?

9. Among 18 persons, there exists 4 persons who know each other, or does not know each other. Show that.

10. Among 6 persons, there exist any least 2 groups of 3 persons, who either know each other, or does not know each other.

11. There are n points on a plane, no three is collinear. Each point is connected with at least k points, $n > k > \frac{n}{2}$, show that there exist at least one triangle.

12. It is given that $2m$ points connected by at least $m^2 + 1$ edges, show that there exists at least one triangle.

13. In a group of 1990 physicists, everyone has worked with at least 1327 physicists, show that there exists a group of 4 physicists, who has worked with each other.

14. In a congregation of 500 participants, each participant knows exactly 400 other participant, can we find 6 participants who know each other? What happen if every participant know more than 400 participants?

15. If a graph G has n vertices, but contains no triangles nor quadrilaterals, then G has at most $\frac{1}{2}n\sqrt{n-1}$ edges.

16. A simple graph isomorphic to its complement is a <u>self-complementary graph</u>. Suppose G is a self-complementary graph with n vertices, show that $n = 4k$ or $n = 4k + 1$, give examples.