IMO Exercises (June 1999)

1. Denote a_n the first nonzero digit of $n!$, thus $a_1 = 1$, $a_2 = 2$, $a_3 = 6$, $a_4 = 4$, $a_5 = 2$, …, is the sequence a_n eventually periodic?

2. *A* and *B* write down positive integers *a* and *b* and give them to the judge. The judge then write down two numbers *x* and *y*, one of them is the sum of the two numbers. The judge then asks *A* : "Do you know the number that another fellow has written down?" If *A* answer no, then the judge will ask *B* the same question, and if *B* answer no, then the judge will ask *A* again, and so on. Assume both *A* and *B* are intelligent and honest, show that in finite steps one of them will answer yes.

3. Find all solutions to the equation $1 + 3^a = 5^b + 3^c$.

4. Find the least number *A* such that for any two squares of combined area 1, a rectangle of area *A* exists such that the two areas may be packed into that rectangle (without the interiors of the squares overlapping), assume the sides of the squares are parallel to the sides of the rectangle.

5. Suppose each of the twenty students may choose zero to six subjects from six subjects offered. Is it true that there are five students and two subjects such that all five have chosen subjects of have chosen neither?

6. Let *A*, *B* and *C* be vertices of triangle inscribed in a unit circle and *P* a point inside the triangle. Show that

$$PA \cdot PB \cdot PC < \frac{32}{27}.$$