

Practice Problem Set #1 for 1999 further Training

1. Find all integral solutions of the system of equations

$$x + y + z = 3 \text{ and } x^3 + y^3 + z^3 = 3.$$

2. Let the inscribed circle of triangle ABC touch side BC at D , side CA at E and side AB at F . Let G be the foot of perpendicular from D to EF . Show that

$$\frac{FG}{EG} = \frac{BF}{CE}.$$

3. Prove that every integer greater than 17 can be represented as a sum of three integers greater than 1 which are pairwise relatively prime, and show that 17 does not have this property.
4. A circle passing through vertices B and C of triangle ABC intersects sides AB and AC at C' and B' , respectively. Prove that BB' , CC' and HH' are concurrent, where H and H' are the orthocenters of triangles ABC and $A'B'C'$, respectively.

5. (Proposed by Leung Wing Chung) For each positive integer n , define the sequence $a_1 = n$, $a_{i+1} = a_i - 2\sqrt{a_i} + 1$ for $i = 1, 2, 3, \dots$. Find all those n such that $\frac{a_{k+1}}{a_k}$ is rational for some k .