Practice Problem Set #2 for 1999 Further Training

1. Find all integers $n > 1$ such that $1^n + 2^n + \cdots + (n-1)^n$ is divisible by *n*.

2. Let *ABCD* be a cyclic quadrilateral. Let *E* and *F* be variable points on the sides *AB* and *CD*, respectively, such that $AE : EB = CF : FD$. Let *P* be the point on the segment *EF* such that $PE : PF = AB : CD$. Prove that the ratio between the areas of triangles *APD* and *BPC* does not depend on the choice of *E* and *F*.

3. Let $a_1, a_2, \cdots, a_n$ be distinct positive integers. Prove that

$$(a_1^7 + a_2^7 + \cdots + a_n^7) + (a_1^5 + a_2^5 + \cdots + a_n^5) \geq 2(a_1^3 + a_2^3 + \cdots + a_n^3)^2 .$$

   Can equality occur？

4. Show that if $p > 3$ is prime, then p^n cannot be the sum of two positive cubes for any $n \geq 1$. What about $p = 2$ or 3？

5. Let *ABCDEF* be a convex hexagon such that

$$\angle B + \angle D + \angle F = 360° \text{ and } \frac{AB}{BC} \cdot \frac{CD}{DE} \cdot \frac{EF}{FA} = 1 .$$

   Prove that

$$\frac{BC}{CA} \cdot \frac{AE}{EF} \cdot \frac{FD}{DB} = 1 .$$

Remark：Problem 2 and Problem 5 are unused problems of 1998 IMO.