

Practice Problem Set B for 1999 Further Training

1. Let n, t denote non-negative integers. Can integers $4^n(8t+7)$ be written as sum of 3 squares? What about positive integers not of the form $4^n(8t+7)$?
2. Find all integral solutions of $x^2 + y^2 + z^2 = x^2y^2$.
3. Show that if $a_1a_2 + a_2a_3 + \cdots + a_{n-1}a_n + a_na_1 = 0$ with $a_i \in \{1, -1\}$, then $4 \mid n$.
4. Show that the sequence $a_n = \sqrt{24n+1}$, where n is positive integer, contains all primes except 2 and 3.
5. Let n be a positive integer. Prove that if $2 + 2\sqrt{28n^2 + 1}$ is an integer, then it is a square. Can you find all such squares? (or at least the first 5 such squares, in the usual increasing order.)