

Mathematics and Physics

Useful Formulae

1 Pure Mathematics

1.1 Algebra

Fundamental Theorem of Algebra

$P(z)$ has n roots, such that each root is in the complex plane, if $P(z)$ has degree n . For a quadratic, the roots are:

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In a quadratic, if the roots are α and β , then

$$\alpha + \beta = -\frac{b}{a} \quad \alpha\beta = \frac{c}{a}$$

De Moivre's Theorem

$$\forall n \in \mathbb{Q}, \quad (a + bi)^n = [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Properties of Complex Numbers

$\forall z, w \in \mathbb{C}$,

$$\begin{aligned} \bar{\bar{z}} &= z \\ z &= \bar{\bar{z}} \text{ if } z \in \mathbb{R} \\ z + \bar{z} &= 2\text{Re}(z) \\ |z| &= |\bar{z}| \\ z\bar{z} &= |z|^2 \\ \overline{(z+w)} &= \bar{z} + \bar{w} \\ \overline{(zw)} &= \bar{z}\bar{w} \\ \text{Re}(z) &< |z| \\ |z+w| &\leq |z| + |w| \end{aligned}$$

Binomial Theorem

$$(a + b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r, \quad n \in \mathbb{N}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots, \quad n \in \mathbb{Q}$$

1.2 Trigonometric Identities

$$\begin{aligned} \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta} \end{aligned}$$

Table 1 Standard Integrals and Derivatives

$f(x)$	$f'(x)$	$\int f(x) dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1} + C$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln \sec x + C$
$\sec^2 x$		$\tan x + C$
$\sec x$	$\sec x \tan x$	$\ln \sec x + \tan x + C$
$\text{cosec } x$	$-\text{cosec } x \cot x$	$\ln \left \tan \frac{1}{2}x \right + C$
$\cot x$	$-\text{cosec}^2 x$	$\ln \sin x + C$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	
	$\frac{1}{x}$	$-\frac{1}{x^2}$
		$\ln x + C$
$f(g)$	$f'(g)g'$	
fg	$f'g + fg'$	
$\frac{f}{g}$	$\frac{f'g - fg'}{g^2}$	
$\sin^{-1} \left(\frac{x}{a} \right)$	$\frac{1}{\sqrt{a^2 - x^2}}$	
$\cos^{-1} \left(\frac{x}{a} \right)$	$\frac{-1}{\sqrt{a^2 - x^2}}$	
$\tan^{-1} \left(\frac{x}{a} \right)$	$\frac{1}{a^2 + x^2}$	
	$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$

$$\begin{aligned} 2 \sin \alpha \cos \beta &\equiv \sin(\alpha + \beta) + \sin(\alpha - \beta) \\ 2 \cos \alpha \sin \beta &\equiv \sin(\alpha + \beta) - \sin(\alpha - \beta) \\ 2 \cos \alpha \cos \beta &\equiv \cos(\alpha + \beta) + \cos(\alpha - \beta) \\ -2 \sin \alpha \sin \beta &\equiv \cos(\alpha + \beta) - \cos(\alpha - \beta) \end{aligned}$$

$$\begin{aligned} \sin \theta + \sin \phi &\equiv 2 \sin \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi) \\ \sin \theta - \sin \phi &\equiv 2 \cos \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi) \\ \cos \theta + \cos \phi &\equiv 2 \cos \frac{1}{2}(\theta + \phi) \cos \frac{1}{2}(\theta - \phi) \\ \cos \theta - \cos \phi &\equiv -2 \sin \frac{1}{2}(\theta + \phi) \sin \frac{1}{2}(\theta - \phi) \end{aligned}$$

$$\begin{aligned} \sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \end{aligned}$$

Table 2 Values of Common Functions

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	x	$\log_a x$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	undefined
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	1	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	a	1

$$\begin{aligned}\sin^2 \alpha &= \frac{1}{2}(1 - \cos 2\alpha) \\ &= \frac{1}{2}(3 - \sin 2\alpha) \\ &= 1 - \cos^2 \alpha \\ \cos^2 \alpha &= \frac{1}{2}(\sin 2\alpha - 1) \\ &= \frac{1}{2}(1 + \cos 2\alpha) \\ \sec^2 \alpha &= 1 + \tan^2 \alpha \\ \operatorname{cosec}^2 \alpha &= 1 + \cot^2 \alpha\end{aligned}$$

$$\begin{aligned}\sin 3\alpha &= 3 \sin \alpha - 4 \sin^3 \alpha \\ \cos 3\alpha &= 4 \cos^3 \alpha - 3 \cos \alpha \\ \tan 3\alpha &= \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}\end{aligned}$$

Hyperbolic Functions

$$\begin{aligned}\sinh(x) &= \frac{e^x - e^{-x}}{2} = -i \sin(ix) \\ \cosh(x) &= \frac{e^x + e^{-x}}{2} = \cos(ix) \\ \tanh(x) &= \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{\sinh(x)}{\cosh(x)}\end{aligned}$$

1.3 Vectors

Magnitude of Vector \mathbf{a} :

$$|\mathbf{a}| = \sqrt{p^2 + q^2 + r^2}$$

where $\mathbf{a} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$.

Unit Vector:

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

Scalar Product:

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= |\mathbf{a}| |\mathbf{b}| \cos \theta \\ (x_1, y_1, z_1) \cdot (x_2, y_2, z_2) &= x_1x_2 + y_1y_2 + z_1z_2\end{aligned}$$

Vector Product

For two vectors \mathbf{a} and \mathbf{b} , the vector product is notated as

$$\mathbf{a} \wedge \mathbf{b}$$

and is calculated from

$$(a_2b_3 - a_3b_2)\mathbf{i} - (a_1b_3 - a_3b_1)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$$

It can also be obtained from:

$$\mathbf{a} \wedge \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{u}}$$

where $\hat{\mathbf{u}}$ is the unit vector perpendicular to the plane of \mathbf{a} and \mathbf{b} . The Vector Product is the determinant of

$$\begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}$$

Scalar Triple Product:

$$\mathbf{a} \cdot (\mathbf{b} \wedge \mathbf{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

1.4 Equations of Lines and Planes

Vector equation of a line:

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$$

where \mathbf{a} is a known point on the line, and \mathbf{d} is a direction vector.

Parametric equations of a line:

$$\begin{aligned}x &= a_i + \lambda d_i \\ y &= a_j + \lambda d_j \\ z &= a_k + \lambda d_k\end{aligned}$$

Symmetric form of the equation of a line:

$$\frac{x - a_i}{d_i} = \frac{y - a_j}{d_j} = \frac{z - a_k}{d_k} = \lambda$$

Vector equation of a plane:

$$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$$

where \mathbf{n} is the normal to the plane, and \mathbf{a} is a known point on the plane.

Cartesian form of the equation of a plane, for a normal vector $n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$:

$$xn_1 + yn_2 + zn_3 = d$$

where $d = \mathbf{a} \cdot \mathbf{n}$, and \mathbf{a} is the position vector of a known point on the plane.

Parametric form of equation of plane:

$$\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$$

where \mathbf{a} is the position vector of a point on the plane, and \mathbf{b} and \mathbf{c} are non parallel vectors in the plane.

1.5 Matrices

Determinant of 2×2 matrix:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Inverse of a matrix:

$$A^{-1} = \frac{1}{\text{Det}(A)} \cdot \text{Adj}(A)$$

where $\text{Adj}(A)$ returns the adjugate matrix, the transpose of the matrix of cofactors. For a 2×2 matrix, the inverse is given by:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\text{Det}A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

1.6 Geometry

Circles

Equation of a circle, centre (a, b) and radius r :

$$(x - a)^2 + (y - b)^2 = r^2$$

General Equation of a Circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

where the centre is $(-g, -f)$ and the radius is

$$\sqrt{g^2 + f^2 - c}$$

Cosine Rule

In a triangle ABC:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$$

where r is the radius of the circumcircle of $\triangle ABC$.

1.7 Laws of Logarithms

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

1.8 Sequences and Series

An arithmetic sequence is one of the form:

$$a, a + d, a + 2d, \dots, a + (n - 1)d, \dots$$

where d is the common difference. A geometric sequence takes the form

$$a, ar, ar^2, \dots, ar^{n-1}, \dots$$

where r is the common ratio $\frac{U_n}{U_{n-1}}$.

Sum to n terms of an arithmetic series:

$$S_n = \frac{1}{2}n [2a + (n - 1)d]$$

Sum to n terms of a geometric series:

$$S_n = \frac{a(1 - r^n)}{1 - r}, n \in \mathbb{N}$$

where a is the first term in the sequence. Also,

$$S_\infty = \frac{a}{1 - r} \text{ if } |r| < 1$$

General Expansions

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 \dots, |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^{ix} = \cos x + i \sin x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n = e$$

$$\lim_{n \rightarrow \infty} \left[n \tan \left(\frac{180}{n} \right) \right] = \pi$$

$$\sum_{r=1}^n r = \frac{1}{2}n(n + 1), n \in \mathbb{N}$$

$$\sum_{r=0}^n r^2 = \frac{1}{6}n(n + 1)(2n + 1)$$

Maclaurin Expansion

The Maclaurin series for a function $f(x)$ is

$$\begin{aligned} f(x) &= f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + \dots + f^{(n)}(0)\frac{x^n}{n!} \\ &= \sum_{r=0}^n f^{(r)}(0)\frac{x^r}{r!} \end{aligned}$$

1.9 Differential Equations

First Order, Linear

An equation of the form

$$\frac{dy}{dx} + P(x)y = f(x)$$

can be solved using

$$e^{\int P(x) dx} y = \int e^{\int P(x) dx} f(x) dx$$

Second Order, Linear, Homogeneous

For an equation

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

there is an auxiliary equation

$$a\omega^2 + b\omega + c = 0$$

- For real and distinct roots for ω

$$y = Ae^{\omega_1 x} + Be^{\omega_2 x}$$

- For real and equal roots for ω

$$y = (A + Bx)e^{\omega x}$$

- For complex roots $p \pm iq$

$$y = e^{px}(A \cos qx + B \sin qx)$$

2 Greek Alphabet

See Table 4 on Page 10.

3 Applied Mathematics: Mechanics

3.1 Basic Kinematics

Let displacement equal s , velocity equal v , and acceleration equal a . Alternatively, displacement is x , velocity \dot{x} , and acceleration \ddot{x} .

$$\begin{aligned} v &= \frac{ds}{dt} = \frac{d}{dt} \left(ut + \frac{1}{2}at^2 \right) \\ a &= \frac{dv}{dt} = \frac{d}{dt} (u + at) \\ a &= \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx} \end{aligned}$$

3.2 Projectiles

For a projectile, the displacement \mathbf{r} , in two dimensions, is defined as:

$$\mathbf{r} = \left(ut \cos \alpha, ut \sin \alpha - \frac{1}{2}gt^2 \right)$$

where \mathbf{v} , the velocity, is defined as

$$\mathbf{v} = (u \cos \alpha, u \sin \alpha - gt)$$

An equation in x and y for the trajectory is given as:

$$\begin{aligned} y &= x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha} \\ &= x \tan \alpha - \frac{g}{2u^2} x^2 \sec^2 \alpha \end{aligned}$$

The time of flight of a projectile:

$$t = \frac{2u \sin \alpha}{g}$$

The horizontal range for a horizontal plane:

$$x = \frac{u^2 \sin 2\alpha}{g}$$

Maximum height of projectile:

$$y = \frac{u^2 \sin^2 \alpha}{2g}$$

3.3 Friction

For an object on the point of slipping:

$$F = \mu R$$

where μ is the coefficient of friction. The coefficient of friction is given by:

$$\mu = \tan \lambda$$

where λ is the angle of friction. (i.e. The angle of the slope on the point of the object slipping.)

3.4 Oscillations & S.H.M.

Hooke's Law:

$$T = kx = \frac{\lambda x}{l}$$

where k is the stiffness constant, l the natural length, and λ the modulus of elasticity.

Equations for S.H.M.:

$$\begin{aligned} \ddot{x} &= -\omega^2 x \\ v^2 &= \omega^2 (a^2 - x^2) \\ x &= a \sin(\omega t + \alpha) \end{aligned}$$

where a is the amplitude and α is the phase of the oscillations.

Maximum Speed and Acceleration:

$$\begin{aligned}v_{max} &= a\omega \\ \dot{x}_{max} &= a\omega^2\end{aligned}$$

Period:

$$\tau = \frac{2\pi}{\omega}$$

Damped Oscillations

Equation of motion for a damped oscillating system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Basic form of solution:

$$x = Ae^{\omega t}$$

Auxilliary Equation for $[m\ddot{x} + c\dot{x} + kx = 0]$:

$$m\omega^2 + c\omega + k = 0$$

Solutions to Auxilliary Equation:

- If $c^2 > 4mk$ then the negative roots are ω_1 and ω_2 , giving:

$$x = Ae^{\omega_1 t} + Be^{\omega_2 t}$$

- If $c^2 = 4mk$ then the roots are equal, giving:

$$x = (A + Bt)e^{\omega t}$$

$$\text{where } \omega = -\frac{c}{2m}$$

- If $c^2 < 4mk$ the roots are imaginary:

$$\omega = -\frac{c}{2m} \pm in$$

$$\text{where } i = \sqrt{-1} \text{ and } n = \sqrt{\frac{4mk - c^2}{4m^2}}$$

Final Equation of Motion:

$$\begin{aligned}x &= Ce^{-\frac{c}{2m}t} \cos(nt + \phi) \\ &= e^{-\frac{c}{2m}t} [C \cos(nt) + D \sin(nt)]\end{aligned}$$

The trigonometric functions are the result of an infinite expansion for e^x .

3.5 Circular Motion

Unit Vectors:

$$\begin{aligned}\mathbf{e}_r &= \cos \theta \mathbf{i} + \sin \theta \mathbf{j} \\ \mathbf{e}_\theta &= -\sin \theta \mathbf{i} + \cos \theta \mathbf{j}\end{aligned}$$

Displacement of particle moving with Circular Motion:

$$\begin{aligned}\mathbf{r} &= r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j} \\ &= r\mathbf{e}_r\end{aligned}$$

Velocity of particle:

$$\begin{aligned}\mathbf{v} &= -r \sin \theta \frac{d\theta}{dt} \mathbf{i} + r \cos \theta \frac{d\theta}{dt} \mathbf{j} \\ &= r \frac{d\theta}{dt} \mathbf{e}_\theta\end{aligned}$$

Acceleration of Particle:

$$\mathbf{a} = -r \left(\frac{d\theta}{dt} \right)^2 \mathbf{e}_r + r \frac{d^2\theta}{dt^2} \mathbf{e}_\theta$$

Where $\frac{d\theta}{dt}$ is constant

$$\begin{aligned}\mathbf{r} &= r \cos(\omega t) \mathbf{i} + r \sin(\omega t) \mathbf{j} \\ \mathbf{v} &= r\omega [-\sin(\omega t) \mathbf{i} + \cos(\omega t) \mathbf{j}] \\ \mathbf{a} &= -r\omega^2 [\cos(\omega t) \mathbf{i} + \sin(\omega t) \mathbf{j}] = -\omega^2 \mathbf{r}\end{aligned}$$

3.6 Dynamics

The Impulse (and change in momentum) of a force is given by:

$$\int_{t_1}^{t_2} \mathbf{F} dt = m\mathbf{v} - m\mathbf{u}$$

Work done for a constant force:

$$W = \mathbf{F} \cdot \mathbf{d}$$

Work done for a force as a function of x or t :

$$\begin{aligned}W &= \int_{x_1}^{x_2} \mathbf{F}(x) dx \\ &= \int_{t_1}^{t_2} \mathbf{F}(t) \cdot \mathbf{v} dt \text{ where } \mathbf{v} = \frac{dx}{dt} \mathbf{i}\end{aligned}$$

Power is given by:

$$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

4 Physics: Mechanics

4.1 Angular Motion

In angular motion, the angular displacement is represented by θ , angular velocity ω and angular acceleration α . The following table presents angular motion equations as they are analogous to linear motion equations:

Linear	Angular
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v = u + at$	$\omega = \omega_0 + \alpha t$

The period and frequency of an object's rotation:

$$\begin{aligned}T &= \frac{2\pi}{\omega} \\ f &= \frac{1}{T} = \frac{\omega}{2\pi}\end{aligned}$$

Families of angular movement equations:

Tangential Velocity:

$$v = \frac{r\theta}{t} = r\omega$$

Tangential Acceleration:

$$a_{tan} = r\alpha$$

Centripetal Acceleration:

$$a_{cent} = \frac{v^2}{r} = r\omega^2$$

Centripetal Force:

$$F_{cent} = \frac{mv^2}{r} = mr\omega^2$$

4.2 Rotational Dynamics

Torque on an object:

$$T = Fr = I\alpha = mr^2\alpha$$

Moment of Inertia:

$$I = \sum_{i=1}^n m_i r_i^2$$

Angular Momentum L :

$$L = I\omega$$

Rotational Kinetic Energy:

$$E_{rk} = \frac{1}{2}I\omega^2$$

Work Done:

$$E_W = T\theta$$

4.3 Relativistic Dynamics

$$E = mc^2$$

$$mc^2 = T + m_0c^2$$

Relativistic Mass at velocity v :

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

4.4 Simple Harmonic Motion

Simple Harmonic Motion Equation:

$$\frac{d^2y}{dt^2} = -\omega^2y$$

Solutions to the S.H.M Equation:

$$y = A \sin \omega t$$

$$= A \cos \omega t$$

where A is the maximum displacement (amplitude).

Velocity of the particle:

$$v = \pm\omega\sqrt{A^2 - y^2}$$

Kinetic Energy in S.H.M.:

$$E_K = \frac{1}{2}m\omega^2(A^2 - y^2)$$

Potential Energy:

$$E_P = \frac{1}{2}m\omega^2y^2$$

4.5 Gravitation

Force between two masses:

$$F = \frac{Gm_1m_2}{r^2}$$

Gravitational Potential:

$$V = -\frac{Gm}{r}$$

Gravitational Potential Energy:

$$E_p = -\frac{Gm_1m_2}{r}$$

Gravitational Field Strength:

$$g = \frac{Gm}{r^2}$$

4.6 Wave-Particle Duality

Quantisation of Angular Momentum:

$$mvr = n\hbar$$

De Broglie Wavelength:

$$\lambda = \frac{h}{p}$$

5 Physics: Electrical Phenomena

Any E without a subscript refers to the electric field strength, where as a subscript E refers to an energy value. (This applies to Section 5 only.)

5.1 Electric FieldsForce exerted on a charge q , in a field of strength E :

$$F = qE$$

Force between two charges Q_1 and Q_2 , r metres apart:

$$|F| = \frac{Q_1Q_2}{4\pi\epsilon_0r^2}$$

where ϵ_0 is the permittivity of free space.

Electric field strength in uniform field between two plates, d metres apart:

$$E = \frac{V}{d}$$

Electric field strength r metres from charge Q :

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

Electrostatic Potential, r metres from charge Q :

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Electrostatic Potential Energy:

$$E_{ESP} = qV$$

Final Velocity of particle moving through uniform field:

$$v = \sqrt{\frac{2qV}{m}}$$

Distance of closest approach of charge q :

$$r = \frac{2qQ}{4\pi\epsilon_0 mv^2}$$

Acceleration of particle in uniform field:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$$

5.2 Magnetic Fields

Force on current-carrying conductor in a magnetic field:

$$F = BIl \sin \theta = qvB \sin \theta$$

Magnetic induction due to a conductor:

$$B = \frac{\mu_0 I}{2\pi r}$$

where μ_0 is the permeability of free space.

Force between two wires:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

For a charged particle in a magnetic field:

$$qvB = \frac{mv^2}{r}$$

where r is the radius of the circle in which the particle is moving.

For a particle moving in a magnetic field:

$$d = \frac{2\pi r}{\tan \theta}, \quad 0 < \theta < \frac{\pi}{2}$$

where d is the pitch of the Helix.

5.3 Electromagnetic Fields

Velocity of a charge moving perpendicular to E and B fields:

$$v = \frac{E}{B}$$

Relating important physical constants:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

5.4 Self Inductance

E.M.F. of an inductor in a circuit:

$$\epsilon = -L \frac{dI}{dt}$$

Energy in the magnetic field in the inductor:

$$E_E = \frac{1}{2} LI^2$$

Reactance of an inductor:

$$\begin{aligned} X_L &= \frac{V_0}{I_0} \\ &= 2\pi fL \end{aligned}$$

6 Physics: Wave Phenomena

Equation of a travelling wave:

$$y(x, t) = A \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$$

where λ is the wavelength, f the frequency and A the amplitude.

Phase difference ϕ :

$$\phi = \frac{2\pi x}{\lambda}$$

where x is the displacement of one wave relative to the other.

Wavelength of n th harmonic:

$$\lambda_n = \frac{\lambda_1}{n}$$

n th harmonic frequency:

$$f_n = \frac{nv}{\lambda_1} = n f_1$$

The Doppler Effect

For a source moving towards a stationary observer:

$$f' = f \frac{v}{v - v_s}$$

For a source moving away from a stationary observer:

$$f' = f \frac{v}{v + v_s}$$

For an observer moving towards a stationary source:

$$f' = f \frac{v + v_{obs}}{v}$$

For an observer moving away from a stationary source:

$$f' = f \frac{v - v_{obs}}{v}$$

6.1 Interference by Division of Amplitude

The optical path length is defined as nd , where d is the distance travelled, and n the refractive index of the material. For constructive interference,

$$OPD = m\lambda, \quad m \in \mathbb{Z}$$

For destructive interference,

$$d = \left(m + \frac{1}{2}\right) \lambda$$

For an arrangement such that $n_1 < n_3 < n_2$,

$$d = \frac{\left(m - \frac{1}{2}\right) \lambda}{2n}$$

where d is the thickness of the film and $m \in \mathbb{Z}$. For destructive interference, as in the above situation,

$$d = \frac{m\lambda}{2n}$$

For a lens coating,

$$d = \frac{\left(m + \frac{1}{2}\right) \lambda}{2n}$$

The minimum thickness of a lens coating to reduce glare is given by

$$d = \frac{\lambda}{4n}$$

Fringe separation in wedge fringes:

$$\Delta x = \frac{\lambda l}{2ny}$$

where n is the refractive index of the material between the slides.

Fringe separation in Young's Slits:

$$\Delta x = \frac{\lambda D}{d}$$

Malus' Law for polarisation:

$$I = I_0 \cos^2 \theta$$

Brewster's Law for polarisation by reflection:

$$\tan i_p = n$$

where n is the refractive index of the reflective material.

7 Physics: Uncertainties

Calculation	Uncertainty
$X = A \pm B$	$\Delta X = \sqrt{(\Delta A)^2 + (\Delta B)^2}$
$X = AB$ or $X = \frac{A}{B}$	$\frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$
$X = A^n$	$\Delta X = n(\Delta A)$ (ΔA & ΔX are %.)
gradient of line	$\Delta m_{\text{dataline}} = \frac{ m_2 - m_1 }{\sqrt{n - 2}}$
y-intercept	$\Delta c = \frac{ c_2 - c_1 }{\sqrt{n - 2}}$

8 Mathematical & Physical Constants

See Table 5.

Table 3 Mathematical Notation

\in	'is an element of'
\notin	'is not an element of'
\subset	'is a strict subset of'
\subseteq	'is a subset of'
\cup	union
\cap	intersection
$[a, b]$	$x \in \mathbb{R} : a \leq x \leq b$
(a, b)	$x \in \mathbb{R} : a < x < b$
$[a, b)$	$x \in \mathbb{R} : a \leq x < b$
\forall	'for all'
\exists	'there exists'
\Rightarrow	'implies'
\Leftrightarrow	'implies and is implied by' / 'is logically equivalent to'
\therefore	'therefore'
\because	'because'
\neg	negation
\mathbb{N}	The set of natural numbers
\mathbb{W}	The set of whole numbers
\mathbb{Z}	The set of integers
\mathbb{Q}	The set of rational numbers
\mathbb{R}	The set of real numbers
\mathbb{C}	The set of complex numbers
\emptyset	The empty set
\cdot	Scalar Product (vectors and scalars)
\wedge	Vector Product (or Cross Product)
Σ	Sum
∞	Infinity
i	$\sqrt{-1}$

Table 4 Greek Alphabet

<i>A</i>	alpha	α	<i>I</i>	iota	ι	<i>P</i>	rho	ρ
<i>B</i>	beta	β	<i>K</i>	kappa	κ	Σ	sigma	σ
Γ	gamma	γ	Λ	lambda	λ	<i>T</i>	tau	τ
Δ	delta	δ	<i>M</i>	mu	μ	<i>Y</i>	upsilon	υ
<i>E</i>	epsilon	ϵ	<i>N</i>	nu	ν	Φ	phi	ϕ
<i>Z</i>	zeta	ζ	Ξ	xi	ξ	<i>X</i>	chi	χ
<i>H</i>	eta	η	<i>O</i>	omicron	\omicron	Ψ	psi	ψ
Θ	theta	θ	Π	pi	π	Ω	omega	ω

Table 5 Mathematical and Physical Constants and Values

Quantity	Symbol	Value
Acceleration due to gravity on Earth	g	9.8ms^{-2}
Universal Gravitational Constant	G	$6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$
Speed of Light in Vacuum	c	$3 \times 10^8\text{ms}^{-1}$
Speed of Sound in Air	—	$3.4 \times 10^2\text{ms}^{-1}$
Charge on Electron (Magnitude)	e	$1.60 \times 10^{-19}\text{C}$
Mass of Electron	m_e	$9.11 \times 10^{-31}\text{kg}$
Mass of Proton	m_p	$1.673 \times 10^{-27}\text{kg}$
Mass of Neutron	m_n	$1.675 \times 10^{-27}\text{kg}$
Plank's Constant	h	$6.63 \times 10^{-34}\text{Js}$
$h/2\pi$ (Dirac's Constant)	\hbar	$1.0546 \times 10^{-31}\text{Js}$
Permeability of Free Space	μ_0	$4\pi \times 10^{-7}\text{Hm}^{-1}$
Permittivity of Free Space	ϵ_0	$8.85 \times 10^{-12}\text{Fm}^{-1}$
Radius of Earth	—	$6.4 \times 10^6\text{m}$
Mass of Earth	—	$6 \times 10^{24}\text{kg}$
Radius of Moon's Orbit	—	$3.84 \times 10^8\text{m}$
Mass of Moon	—	$7.3 \times 10^{22}\text{kg}$
Pi	π	3.141592
Euler's Number	e	2.718282
Square root of two	$\sqrt{2}$	1.414214