

## Notation

The following notation will be used in all mathematics examinations:

### 1. Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x:\dots\}$	the set of all $x$ such that ...
$n(A)$	the number of elements in set $A$
$\emptyset$	the empty set
$\mathcal{E}$	the universal set
$A'$	the complement of the set $A$
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}_n$	the set of integers modulo $n$ , $\{0, 1, 2, \dots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
$(x, y)$	the ordered pair $x, y$
$A \times B$	the cartesian product of sets $A$ and $B$ , ie $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b), [a, b [$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b], ]a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b), ]a, b [$	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	$y$ is related to $x$ by the relation $R$
$y \sim x$	$y$ is equivalent to $x$ , in the context of some equivalence relation

## 2. Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq, \nlessgtr$	is less than or equal to, is not greater than
$>$	is greater than
$\geq, \ngtr$	is greater than or equal to, is not less than
$\infty$	infinity
$p \wedge q$	$p$ and $q$
$p \vee q$	$p$ or $q$ (or both)
$\sim p$	not $p$
$p \Rightarrow q$	$p$ implies $q$ (if $p$ then $q$ )
$p \Leftarrow q$	$p$ is implied by $q$ (if $q$ then $p$ )
$p \Leftrightarrow q$	$p$ implies and is implied by $q$ ( $p$ is equivalent to $q$ )
$\exists$	there exists
$\forall$	for all

## 3. Operations

$a + b$	$a$ plus $b$
$a - b$	$a$ minus $b$
$a \times b, ab, a.b$	$a$ multiplied by $b$
$a \div b, \frac{a}{b}, a/b$	$a$ divided by $b$
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
$\sqrt{a}$	the positive square root of $a$
$ a $	the modulus of $a$
$n!$	$n$ factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbf{Z}^+$ $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbf{Q}$

## 4. Functions

$f(x)$	the value of the function $f$ at $x$
$f : A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f : x \mapsto y$	the function $f$ maps the element $x$ to the element $y$

$f^{-1}$	the inverse function of the function $f$
$g \circ f, gf$	the composite function of $f$ and $g$ which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
$\Delta x, \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... , $n$ th derivatives of $f(x)$ with respect to $x$
$\int y dx$	the indefinite integral of $y$ with respect to $x$
$\int_a^b y dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of $V$ with respect to $x$
$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to $t$

## 5. Exponential and Logarithmic Functions

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x, \log_e x$	natural logarithm of $x$
$\lg x, \log_{10} x$	logarithm of $x$ to base 10

## 6. Circular and Hyperbolic Functions

$\left. \begin{array}{l} \sin, \cos, \tan, \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \arcsin, \arccos, \arctan, \\ \operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth} \end{array} \right\}$	the inverse hyperbolic functions

## 7. Complex Numbers

$i, j$	square root of $-1$
$z$	a complex number, $z = x + iy$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im} z = y$

$ z $	the modulus of $z$ , $ z  = \sqrt{(x^2 + y^2)}$
$\arg z$	the argument of $z$ , $\arg z = \theta$ , $-\pi < \theta \leq \pi$
$z^*$	the complex conjugate of $z$ , $x - iy$

## 8. Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix $\mathbf{M}$

## 9. Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} , a$	the magnitude of $\mathbf{a}$
$\left  \overrightarrow{AB} \right , AB$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

## 10. Probability and Statistics

$A, B, C$ , etc	events
$A \cup B$	union of the events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$	complement of the event $A$
$P(A   B)$	probability of the event $A$ conditional on the event $B$
$X, Y, R$ , etc	random variables
$x, y, r$ , etc	values of the random variables $X, Y, R$ , etc
$x_1, x_2, \dots$	observations
$f_1, f_2, \dots$	frequencies with which the observations $x_1, x_2, \dots$ occur
$p(x)$	probability function $P(X = x)$ of the discrete random variable $X$
$p_1, p_2, \dots$	probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$
$f(x), g(x), \dots$	the value of the probability density function of a continuous random variable $X$
$F(x), G(x), \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable $X$

$E(X)$	expectation of the random variable $X$
$E[g(X)]$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable $X$
$G(t)$	probability generating function for a random variable which takes the values 0, 1, 2, ...
$B(n, p)$	binomial distribution with parameters $n$ and $p$
$N(\mu, \sigma^2)$	normal distribution with mean $\mu$ and variance $\sigma^2$
$\mu$	population mean
$\sigma^2$	population variance
$\sigma$	population standard deviation
$\bar{x}, m$	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
$\phi$	probability density function of the standardised normal variable with distribution $N(0,1)$
$\Phi$	corresponding cumulative distribution function
$\rho$	product moment correlation coefficient for a population
$r$	product moment correlation coefficient for a sample
$\text{Cov}(X, Y)$	covariance of $X$ and $Y$