



UNIVERSITY OF CAMBRIDGE

Faculty of Mathematics

MATHEMATICAL READING LIST

This list of interesting mathematics books is mainly intended for sixth-formers planning to take a degree in mathematics. However, everyone who likes mathematics should take a look: some of the items are very suitable for less experienced readers and even the most hardened mathematician will probably find something new here.

1 INTRODUCTION

The range of mathematics books now available is enormous. This list just contains a few suggestions which you should find helpful. They are divided into three groups: historical and general (which aim to give a broad idea of the scope and development of the subject); recreational, from problem books (which aim to keep your brain working) to technical books (which give you insight into a specific area of mathematics and include mathematical discussion); and textbooks (which cover a topic in advanced mathematics of the kind that you will encounter in your first year at university). Do not feel that you should only read the difficult ones: medicine is only good for you if it is hard to take, but this is not true for mathematics books. Any reading you do will certainly prove useful.

All the books on the list should be obtainable from your local library, though you may have to order them. Most are available (relatively) cheaply in paperback and so would make good additions to your Christmas list. Some may be out of print, but still obtainable from libraries.

You might also like to look on the web for mathematics sites. Good starting points are:

NRICH (<http://www.nrich.maths.org.uk>), which is a web-based interactive mathematics club;

Plus (<http://plus.maths.org.uk>), which is a web-based mathematics journal.

Both these sites are based in Cambridge.

2 HISTORICAL AND GENERAL

One of the most frequent complaints of mathematics undergraduates is that they did not realise until too late what was behind all the material they wrote down in lectures: Why was it important? What were the problems which demanded this new approach? Who did it? There is much to be learned from a historical approach, even if it is fairly non-mathematical.

Makers of Mathematics S. Hollingdale (Penguin, 1989)

There are not many books on the history of mathematics which are pitched at a suitable level. Hollingdale gives a biographical approach which is both readable and mathematical. You might also try E.T. Bell *Men of Mathematics* (Touchstone Books, Simon and Schuster, 1986). Historians of mathematics have a lot to say about this (very little of it complimentary) but it is full of good stories which have inspired generations of mathematicians.

A Russian Childhood S. Kovalevskaya (trans. B. Stillman) (Springer, 1978, now out of print)

Sonya Kovalevskaya was the first woman in modern times to hold a lectureship at a European university: in 1889, in spite of the fact that she was a woman (with an unconventional private life), a foreigner, a socialist (or worse) and a practitioner of the new Weierstrassian theory of analysis, she was appointed a professor at the University of Stockholm. Her memories of childhood are non-mathematical but fascinating. She discovered in her nursery the theory of infinitesimals: times being hard, the walls had been papered with pages of mathematical notes.

Alan Turing, the Enigma A. Hodges (Vintage, 1992)

A great biography of Alan Turing, a pioneer of modern computing. The title has a double meaning: the man was an enigma, committing suicide in 1954 by eating a poisoned apple, and the German code that he was instrumental in cracking was generated by the Enigma machine. The book is largely non-mathematical, but there are no holds barred when it comes to describing his major achievement, now called a Turing machine, with which he demonstrated that a famous conjecture by Hilbert is false.

The Man Who Knew Infinity R. Kanigel (Abacus, 1992)

The life of Ramanujan, the self-taught mathematical prodigy from a village near Madras. He sent Hardy samples of his work from India, which included rediscoveries of theorems already well known in the West and other results which completely baffled Hardy. Some of his estimates for the number of ways a large integer can be expressed as the sum of integers are extraordinarily accurate, but seem to have been plucked out of thin air.

A Mathematician's Apology G.H. Hardy (CUP, 1992)

Hardy was one of the best mathematicians of the first part of this century. Always an achiever (his New Year resolutions one year included proving the Riemann hypothesis, making 211 not out in the fourth test at the Oval, finding an argument for the non-existence of God which would convince the general public, and murdering Mussolini), he led the renaissance in mathematical analysis in England. Graham Greene knew of no writing (except perhaps Henry James's *Introductory Essays*) which conveys so clearly and with such an absence of fuss the excitement of the creative artist. There is an introduction by C.P. Snow.

Littlewood's Miscellany (edited by B. Bollobas) (CUP, 1986)

This collection, first published in 1953, contains some wonderful insights into the development and lifestyle of a great mathematician as well as numerous anecdotes, mathematical (Lion and Man is excellent) and not-so-mathematical. The latest edition contains several worthwhile additions, including a splendid lecture entitled 'The Mathematician's Art of Work', (as well as various items of interest mainly to those who believe that Trinity Great Court is the centre of the Universe). Thoroughly recommended.

The man who loved only numbers Paul Hoffman (Fourth Estate, 1999)

An excellent biography of Paul Erdős, one of the most prolific mathematicians of all time. Erdős wrote over 1500 papers (about 10 times the normal number for a mathematician) and collaborated with 485 other mathematicians. He had no home; he just descended on colleagues with whom he wanted to work, bringing with him all his belongings in a suitcase. Apart from details of Erdős's life, there is plenty of discussion of the kind of problems (mainly number theory) that he worked on.

Surely You're Joking, Mr Feynman R.P. Feynman (Arrow Books, 1992)

Autobiographical anecdotes from one of the greatest theoretical physicists of the last century, which became an immediate best-seller. You learn about physics, about life and (most puzzling of all) about Feynman. Very amusing and entertaining.

Simon Singh Fermat's Last Theorem (Fourth Estate)

You must read this story of Andrew Wiles's proof of Fermat's Last Theorem, including all sorts of mathematical ideas and anecdotes; there is no better introduction to the world of research mathematics. Singh's later *The Code Book* (Fourth Estate) is not so interesting mathematically, but is still a very good read.

Number J. McLeish (Bloomsbury, 1991)

The development of the theory of numbers, from Babylon to Babbage, written with humour and erudition. Hugely enjoyable.

3 RECREATIONAL

You can find any number of puzzle books in the shops and some which are both instructive and entertaining are listed here. Other books in this section do not attempt to set the reader problems, but to give an appetising introduction to important areas of, or recent advances in, mathematics.

Penrose Tiles to Trapdoor Ciphers M. Gardner (CUP/Math. Assoc. of America, 1997)

Or any other book by Martin Gardner: he has written numerous books in similar style, all excellent. His *Mathematical Puzzles and Diversions* and *More Mathematical Puzzles and Diversions* (both available in Penguin) and *The Unexpected Hanging* (Chicago) are classics.

Game, Set and Math. I. Stewart (Penguin, 1997)

Stewart is one of the best current writers of mathematics (recreational or otherwise). This collection (which includes a calculation which shows why you need only be marginally the better player to win a tennis match — whence the title) was originally written in French: some of the puns seem to have suffered in translation, but the *joie de vivre* shines through. You might also like Stewart's book on Chaos, *Does God Play Dice?* (Penguin, 1990). Excellent writing again but, unlike the chaos books mentioned

below, no colour pictures. The title is a quotation from Einstein, who believed (probably incorrectly) that the answer was no; he thought that theories of physics should be deterministic, unlike quantum mechanics which is probabilistic.

To Infinity and Beyond Eli Maor (Princeton, 1991)

Not much hard mathematics here, but lots of interesting mathematical ideas (prime numbers, irrationals, the continuum hypothesis, Olber's paradox (why is the sky dark at night?) and the expanding universe to name but a few), fascinating history and lavish illustrations. The same author has also written a whole book about one number (*e The Story of a Number*), also published by Princeton (1994), but not yet out in paperback.

4 READABLE MATHEMATICS

Mathematics: a very short introduction Timothy Gowers (OUP, 2002)

Gowers is a Fields Medalist (the Fields medal is the mathematical equivalent of the Nobel prize), so it is not at all surprising that what he writes is worth reading. What is surprising is the ease and charm of his writing. He touches lightly many areas of mathematics, some that will be familiar (Pythagoras) and some that may not be (manifolds) and has something illuminating to say about all of them. The book is small and thin: it will fit in your pocket. You should get it.

The Pleasures of Counting T.W. Körner (CUP, 1996)

A brilliant book. There is something here for anyone interested in mathematics and even the most erudite professional mathematicians will learn something new. Some of the chapters involve very little technical mathematics (the discussion of cholera outbreaks which begins the book, for example) while others require the techniques of a first or second year undergraduate course. However, you can skip through the technical bits and still have an idea what is going on. You will enjoy the account of Braess's paradox (a mathematical demonstration of the result, which we all know to be correct, that building more roads can increase journey times), the explanation of why we should all be called Smith, and the account of the Enigma code-breaking. These are just a few of the topics Körner explains with enviable clarity and humour.

What is Mathematics? R. Courant & H. Robbins (OUP, 1996)

A new edition, revised by Ian Stewart, of a classic. It has chapters on numbers (including ∞), logic, cubics, duality, soapfilms, etc. The subtitle (*An elementary approach to ideas and methods*) is rather optimistic: challenging would be a more appropriate adjective, though interesting or instructive would do equally well. Stewart has resisted the temptation to tamper: he has simply updated where appropriate — for example, he discusses the solution to the four-colour problem and the proof of Fermat's Last Theorem.

From Here to Infinity Ian Stewart (OUP, 1996)

This is a revised version of *Problems in Mathematics* (1987); revised of necessity, as the author says, because some of the problems now have solutions — an indication of the speed at which the frontiers of mathematics are receding. Topics discussed include solving the quintic, colouring, knots, infinitesimals, computability and chaos. In the preface, it is guaranteed that the very least you will get from the book is the understanding that mathematical research is not just a matter of inventing new numbers; what you will in fact get is an idea of what real mathematics is.

What's Happening in the Mathematical Sciences B. Cipra (AMS, 1993, '94, '96, '99)

This really excellent series is published by the American Mathematical Society. It contains low(ish)-level discussions, with lots of pictures and photographs, of some of the most important recent discoveries in mathematics. Volumes 1 and 2 cover recent advances in map-colouring, computer proofs, knot theory, travelling salesmen, and much more. Volume 3 (1995–96) has, among other things, articles on Wiles' proof of Fermat's Last Theorem, the investigation of twin primes which led to the discovery that the Pentium chip was flawed, codes depending on large prime numbers and the Enormous Theorem in group theory (the theorem is small but the proof, in condensed form, runs to 5000 pages). Exciting stuff.

Archimedes' Revenge P. Hoffman (Penguin, 1991)

This is not a difficult read, but it covers some very interesting topics: for example, why democracy is mathematically unsound, Turing machines and travelling salesmen. Remarkably, there is no chapter on chaos.

The Mathematical Experience P.J. Davis & R. Hersh (Penguin, 1990)

This gives a tremendous foretaste of the excitement of discovering mathematics. A classic.

Beyond Numeracy J. A. Paulos (Penguin, 1991)

Bite-sized essays on fractals, game-theory, countability, convergence and much more. It is a sequel to his equally entertaining, but less technical, *Numeracy*.

The Penguin Dictionary of Curious and Interesting Numbers D. Wells (Penguin, 1997)

A brilliant idea. The numbers are listed in order of magnitude with historical and mathematical information. Look up 1729 to see why it is 'among the most famous of all numbers'. Look up 0.7404 ($= \pi/\sqrt{18}$) to discover that this is the density of closely-packed identical spheres in what is believed by many mathematicians (though it was at that time an unproven hypothesis) and is known by all physicists and greengrocers to be the optimal packing. Look up Graham's number (the last one in the book), which is inconceivably big: even written as a tower of powers ($9 \uparrow (9 \uparrow (9 \cdot \cdot \cdot))$) it would take up far more ink than could be made from all the atoms in the universe. It is an upper bound for a quantity in Ramsey theory whose actual value is believed to be about 6. A book for the bathroom to be dipped into at leisure. You might also like Wells's *The Penguin Dictionary of Curious and Interesting Geometry* (Penguin, 1991) which is another book for the bathroom. It is not just obscure theorems about triangles and circles (though there are plenty of them); far-reaching results such as the hairy ball theorem (you can't brush the hair flat everywhere) and fixed point theorems are also discussed.

New Applications of Mathematics C. Bondi (ed.) (Penguin, 1991)

Twelve chapters by different authors, starting with functions and ending with supercomputers. There is material here which many readers will already understand, but treated from a novel point of view, and plenty of less familiar but still very understandable material.

Reaching for Infinity S. Gibilisco (Tab/McGraw-Hill, 1990)

A short and comfortable, though mathematical, read about different sorts of infinity. It has theorems, too, which are good for you. An example: $\aleph_0 + \aleph_1 = \aleph_1$. This probably needs a bit of explanation. Loosely speaking: \aleph_0 (pronounced 'aleph' zero) is the number of integers (which is the same as the number of rational numbers) and \aleph_1 is the next biggest infinity. There is another infinity, $c = 2^{\aleph_0}$, which is the number of real numbers. The continuum hypothesis says that $c = \aleph_1$, but it was not realised until 1963 that this cannot be proved or disproved.

The New Scientist Guide to Chaos N. Hall (ed.) (Penguin, 1991)

This comprises a series of articles on various aspects of chaotic systems together with some really amazing photographs of computer-generated landscapes. Chaos is what happens when the behaviour of a system gets too complicated to predict; the most familiar example is the weather, which apparently cannot be forecast accurately more than five days ahead. The articles here delve into many diverse systems in which chaos can occur and include a piece by the guru (Mandelbrot) and one about the mysterious new constant of nature discovered by Feigenbaum associated with the timescale over which dynamical systems change in character.

Chaos J. Gleick (Minerva/Random House, 1997)

Sometimes, at interview, candidates are asked whether they have read any good mathematics books recently. There was a time when nine out of ten candidates who expressed a view named this one. Before that, it was Douglas Hofstadter's *Gödel, Escher, Bach* (Penguin, 1980). Surely they couldn't all have been wrong?

Fractals. Images of Chaos H. Lauwerier (Penguin, 1991)

Poincaré recurrence, Julia sets, Mandelbrot, snowflakes, the coastline of Norway, nice pictures; in fact, just what you would expect to find. But this has quite a bit of mathematics in it and also a number of programs in basic so that you can build your own fractals. It is written with the energy of a true enthusiast.

5 READABLE THEORETICAL PHYSICS

Hidden Unity in Nature's Laws John C. Taylor (CUP, 2001)

When I asked John Taylor which areas of physics his book covered, he said 'Well, all areas'. Having now read it, I see this is more or less true. He takes us from the oldest ideas in physics (about astronomy) to the most modern (string theory). The book is obviously written for an intelligent and interested adult: difficult concepts are not swept under the carpet (there is a chapter on Least Action) and the text is not littered with trendy pictures or jokes. Everything is explained with exceptional clarity in a most engaging manner – almost as if the author was conversing with the reader as an equal.

QED: The Strange Story of Light and Matter R.P. Feynman (Penguin, 1990)

Feynman again, this time explaining the exceedingly deep theory of Quantum Electrodynamics, which describes the interactions between light and electrons, in four lectures to a non-specialist audience – with remarkable success. The theory is not only very strange, it is also very accurate: its prediction of the magnetic moment of the electron agrees with the experimental value to an accuracy equivalent to the width of a human hair in the distance from New York to Los Angeles.

The Cosmic Onion Frank Close (Heinemann, 1983)

Not a great deal has changed on the elementary particle scene since this absorbing survey was written: it was just in time to report first sightings of the Z and W particles. It even reports, with (as it turned out) well-founded scepticism on claims to have seen the top quark. The final chapter makes the all-important link between particle physics (physics on the smallest scale) and cosmology (physics on the largest scale). The energies required to study the latest batch of elementary particles are so great that the Big Bang is the only feasible 'laboratory'.

The Quantum Universe T. Hey & P. Walters (CUP, 1987)

All you ever wanted to know about quantum mechanics, from fusion to fission, from Feynman diagrams to superfluids, and from Higgs particles to Hawking radiation. With potted biographies, historical background, and packed with wonderful illustrations and photographs (including an electron microscope image of a midge). This is an excellent and unusual introduction to the subject. The same authors also wrote a splendid book on relativity (*Einstein's Mirror*).

Was Einstein Right? C.M. Will (Basic Books, 1988)

Einstein's theory of General Relativity is a theory of gravitation which supersedes Newton's theory and is consistent with Special Relativity. The basic idea is that space-time is curved and you feel gravitational forces when you go round a curve in space, in the same way as you feel centrifugal force when your car goes round a bend. This book is about observational tests of the theory, all of which have been passed with flying colours. In particular, there is a binary pulsar which loses mass by gravitational radiation and, as a result, its period of rotation increases by 76 ± 2 millionths of a second per year; General Relativity predicts 75. There is much to be learned here about physics, cosmology and astronomy as well as about Einstein and his theory.

The Accidental Universe P.C.W. Davies (CUP, 1982)

All the buzz-words are here: cosmic dynamics; galactic structure; entropy of the Universe; black holes; many worlds interpretation of quantum mechanics, but this is not another journalistic pot-boiler. It is a careful and accurate account by one of the best writers of popular science.

6 READABLE TEXTBOOKS

There is not much point in trying to cover a lot of material from the first year undergraduate mathematics course you are just about to start, but there is a great deal of point in trying to familiarise yourself with the sort of topics you are going to encounter. It is also a good plan to get used to working on your own; reading mathematics text books is an art not much practised in schools. Many of the following have exercises and answers and some have solutions.

Advanced Problems in Mathematics S.T.C. Siklos (1996 and 2003)

These are selections of STEP-like problems complete with discussion and full solutions. (STEP is the examination normally used as a basis for conditional offers to Cambridge.) The problems are different from most A-level questions, being much longer ('multi-step' is the current terminology) and sometimes covering material from apparently unconnected areas of mathematics. They are more like the sort of problems that you encounter in a university mathematics course, although they are based on the syllabuses of school mathematics. Working through one or both of these booklets would be an excellent way of getting your mathematics up to speed again after the summer break.

The earlier booklet is produced by the examination board which administers STEP: price about £4.50. To obtain copies, write to OCR Publications, P.O. Box 5050, Annersley, Nottingham, NG15 0DL.

The 2003 booklet is in a sense a prequel, since it is based on a less advanced syllabus (basically the A-level core plus some mechanics and probability). It can be downloaded from the our website (www.maths.cam.ac.uk/undergrad/advancedproblems) or obtained by sending an A4 self-addressed envelope and cheque for five pounds ('University of Cambridge') to Faculty Office, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA.

Mathematical Methods for Science Students G. Stephenson (Longman, 1973)

This starts with material you already know and advances cautiously in traditional directions. You may not be bowled over with excitement but you will appreciate the careful explanations, the many examples and exercises and the generally sympathetic approach. You may prefer an entirely problems-based approach, in which case *Worked examples in Mathematics for Scientists and Engineers* (Longman, 1985) by the same author is for you.

Mathematical Methods for Physics and Engineering K F Riley, M P Hobson & S J Bence (Cambridge University Press 1998)

Most of A-level pure mathematics consists of what could be called 'mathematical methods' — i.e. techniques you can use in other areas (such as mechanics and statistics). The continuation of this material forms a basic part of every university course (and would count as applied mathematics!). This book is a strong recommendation for any such course.

A Concise Introduction to Pure Mathematics Martin Liebeck (Chapman & Hall/CRC Mathematics)

This is really excellent. Liebeck provides a simple, nicely explained, appetizer to a wide variety of topics (such as number systems, complex numbers, prime factorisation, number theory, infinities) that would be found in any first year course. His approach is rigorous but he stops before the reader can get too bogged down in detail. There are worked examples (e.g. 'Between any two real numbers there is an irrational') and exercises, which have the same light touch as the text.

What is Mathematical Analysis? John Baylis (MacMillan, 1991)

This book (now out of print, but available from libraries) is part of a series which is supposed to bridge the gap between school and university. It covers some serious analysis (the intermediate value theorem, limits, differentiation and integration) in a most accessible style: it never gets hard, though you will need to study carefully. The layout could be nicer, but do not be put off: you can discover here a function which is continuous at every irrational number but discontinuous (it jumps) at every rational number. Who would have thought that was possible?

Groups: A Path to Geometry R.P. Burn (CUP, 1987)

Permutations, groups, matrices, complex numbers and, above all (or rather, behind all), geometry.

Yet Another Introduction to Analysis V. Bryant (CUP, 1990)

Yes, another; but a very good one. And it has solutions to the many problems. Analysis is the study of all those things you think you already know how to do (such as differentiation, integration), from first principles. This book goes through functions, continuity, series and calculus at a brisk trot; essential material for any mathematician.

A First Course in Mechanics Mary Lunn (OUP, 1991)

A bridge between the sort of mechanics you meet at A-level and the sort you are going to meet at university; not just a bridge, but also a good bit of road on the far side.

Probability and Statistics M.R. Spiegel (Schaum's outline series; McGraw-Hill, 1982)

Part of a large series of mathematics texts which are almost entirely problem based, and consequently are very suitable for home study.

Algorithmics — The Spirit of Computing D. Harel (Addison-Wesley, 1992)

The aim is to impart a deep understanding of the fundamentals of machine-executable processes, and the recipes (algorithms) which govern them. Questions addressed include: 'What problems can be solved by mechanical processes?' and 'What is the minimum cost of obtaining the answer to a given problem?'. The last chapter is about artificial intelligence.