Series

The series of a sequence is the sum of the sequence to a certain number of terms. It is often written as S_n. So if the sequence is 2, 4, 6, 8, 10, ... , the sum to 3 terms = $S_3 = 2 + 4 + 6 = 12$.

The Sigma Notation

The Greek capital sigma, written Σ, is usually used to represent the sum of a sequence. This is best explained using an example:

$$\sum_{r=1}^{4} 3r$$

This means replace the r in the expression by 1 and write down what you get. Then replace r by 2 and write down what you get. Keep doing this until you get to 4, since this is the number above the Σ. Now add up all of the term that you have written down.
This sum is therefore equal to $3\times1 + 3\times2 + 3\times3 + 3\times4 = 3 + 6 + 9 + 12 = 30$.

$$\sum_{r=1}^{3} 3r + 2$$

This is equal to:
$(3\times1 + 2) + (3\times2 + 2) + (3\times3 + 2) = 24$.

The General Case

$$\sum_{r=1}^{n} U_r$$

This is the general case. For the sequence U_r, this means the sum of the terms obtained by substituting in 1, 2, 3,... up to and including n in turn for r in U_r. In the above example, $U_r = 3r + 2$ and $n = 3$.

Arithmetic Progressions

An arithmetic progression is a sequence where each term is a certain number larger than the previous term. The terms in the sequence are said to increase by a common difference, d. For example: 3, 5, 7, 9, 11, is an arithmetic progression where d = 2. The nth term of this sequence is 2n + 1 .
In general, the nth term of an arithmetic progression, with first term a and common difference d, is: a + (n - 1)d . So for the sequence 3, 5, 7, 9, ... $U_n = 3 + 2(n - 1) = 2n + 1$, which we already knew.

The sum to n terms of an arithmetic progression

- $S_n = \frac{1}{2} n [2a + (n - 1)d]$

Example

Sum the first 20 terms of the sequence: 1, 3, 5, 7, 9, ... (i.e. the first 20 odd numbers).

$S_{20} = \frac{1}{2} (20) [2 \times 1 + (20 - 1)\times 2]$
$= 10[2 + 19 \times 2]$
$= 10[40]$
$= \underline{400}$

Geometric Progressions

A geometric progression is a sequence where each term is r times larger than the previous term. r is known as the common ratio of the sequence. The nth term of a geometric progression, where a is the first term and r is the common ratio, is:
ar^{n-1}

For example, in the following geometric progression, the first term is 1, and the common ratio is 2:
1, 2, 4, 8, 16, ...

The nth term is therefore 2^{n-1}

The sum of a geometric progression

The sum of the first n terms of a geometric progression is:

- $\frac{a(1 - r^n)}{1 - r}$

Example

What is the sum of the first 5 terms of the following geometric progression: 2, 4, 8, 16, 32 ?

$S5 = \frac{2(1 - 2^5)}{1 - 2}$

$= \frac{2(1 - 32)}{-1}$

$= \underline{62}$

The sum to infinity of a geometric progression

In geometric progressions where |r| < 1 (in other words where r is less than 1 and greater than –1), the sum of the sequence as n tends to infinity approaches a value. In other words, if you keep adding together the terms of the sequence forever, you will get a finite value. This value is equal to:

- $\frac{a}{1 - r}$

Example

Find the sum to infinity of the following sequence:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \ldots$

Here, a = 1/2 and r = 1/2

Therefore, the sum to infinity is 0.5/0.5 = 1 .

So every time you add another term to the above sequence, the result gets closer and closer to 1.

Harder Example

The first, second and fifth terms of an arithmetic progression are the first three terms of a geometric progression. The third term of the arithmetic progression is 5. Find the 2 possible values for the fourth term of the geometric progression.

The first term of the arithmetic progression is: a

The second term is: $a + d$
The fifth term is: $a + 4d$
So the first three terms of the geometric progression are a, $a + d$ and $a + 4d$.

In a geometric progression, there is a common ratio. So the ratio of the second term to the first term is equal to the ratio of the third term to the second term. So:

$$\frac{a + d}{a} = \frac{a + 4d}{a + d}$$

$(a + d)(a + d) = a(a + 4d)$
$a^2 + 2ad + d^2 = a^2 + 4ad$
$d^2 - 2ad = 0$
$d(d - 2a) = 0$
therefore $d = 0$ or $d = 2a$

The common ratio of the geometric progression, r, is equal to $(a + d)/a$
Therefore, if $d = 0$, $r = 1$
If $d = 2a$, $r = 3a/a = 3$
So the common ratio of the geometric progression is either 1 or 3 .

We are told that the third term of the arithmetic progression is 5. So $a + 2d = 5$. Therefore, when $d = 0$, $a = 5$ and when $d = 2a$, $a = 1$.
So the first term of the arithmetic progression (which is equal to the first term of the geometric progression) is either 5 or 1.

Therefore, when $d = 0$, $a = 5$ and $r = 1$. In this case, the geometric progression is 5, 5, 5, 5, ... and so the fourth term is 5.When $d = 2a$, $r = 3$ and $a = 1$, so the geometric progression is 1, 3, 9, 27, ... and so the fourth term is 27.

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Set Theory

A set is a group of objects. Each object is known as a member of the set. A set can be represented using curly brackets. So a set containing the numbers 2, 4, 6, 8, 10, ... is: {2, 4, 6, 8, 10, ... } . Sets are often also represented by letters, so this set might be *E* = {2, 4, 6, 8, 10, ...} . Alternatively, *E* = {even numbers} .

Common Sets

Some sets are commonly used and so have special notation:

$\mathbb{Z}$ **The set of integers**

$\mathbb{N}$ **The set of natural numbers (positive integers)**

$\mathbb{Q}$ **The set of rational numbers**

$\mathbb{R}$ **The set of real numbers**

For example:

$x \in \mathbb{Z}$

means that x is in the set of integers, i.e. x is an integer.

$\in$ **means 'is a member of'**

Other Notation

Subsets

If A is a subset of B, then all of the elements of A are also in B. For example, if A = {1, 2, 3} and B = {1, 2, 3, 4, 5} then $A \subseteq B$ ($\subseteq$ means is a subset of).

Number of Members

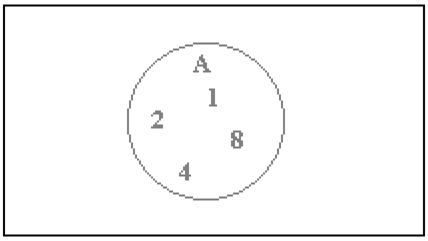
If *A* = {1, 2, 4, 8}, then n(*A*) = 4. This is because n(*A*) means the number of members in set *A*.

The Universal Set

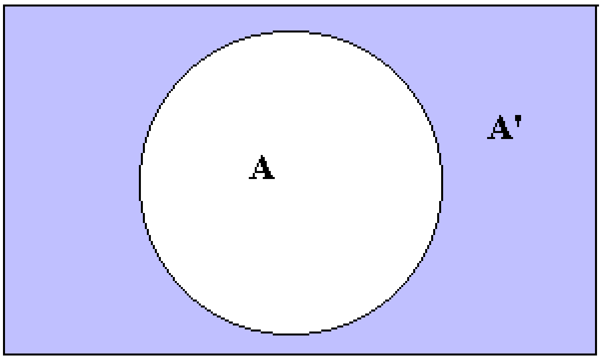
The **universal set** is the set of all sets. All sets are therefore **subsets** of the universal set.

Venn Diagrams

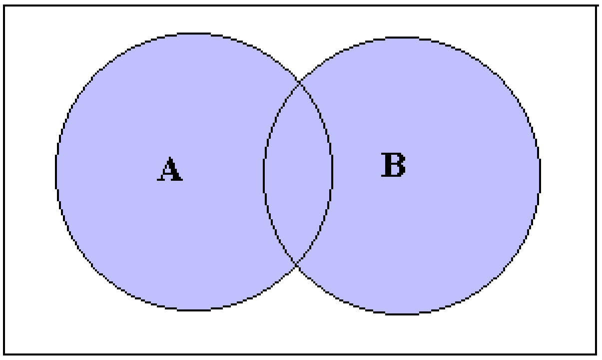
Venn diagrams are used to represent sets. Here, the set *A*{1, 2, 4, 8} is shown using a circle. In Venn diagrams, sets are usually represented using circles. The universal set is the rectangle. The set *A* is a subset of the universal set and so it is within the rectangle.



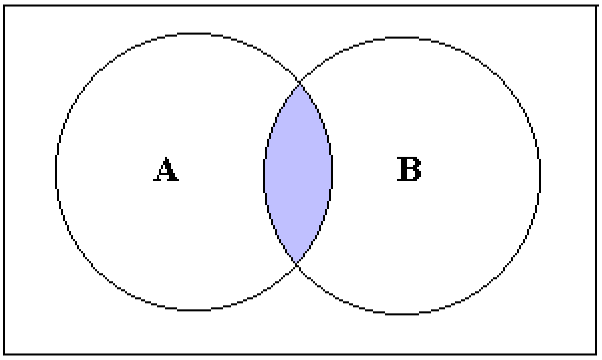
The **complement** of A, written A', contains all events in the sample space which are not members of A. A and A' together cover every possible eventuality.



A ∪ B means the **union** of sets A and B and contains all of the elements of both A and B. This can be represented on a Venn Diagram as follows:



A∩B means the **intersection** of sets A and B. This contains all of the elements which are in both A and B. A∩B is shown on the Venn Diagram below:



An important result connecting the number of members in sets and their unions and intersections is:

- **$n(A) + n(B) - n(A \cap B) = n(A \cup B)$**

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The Binomial Series

Pascal's Triangle

You should know that $(a + b)^2 = a^2 + 2ab + b^2$ and you should be able to work out that $(a + b)^3 = a^3 + 3a^2b + 3b^2a + b^3$.
It should also be obvious to you that $(a + b)^1 = a + b$.

so $(a + b)^1 = a + b$
$(a + b)^2 = a^2 + 2ab + b^2$
$(a + b)^3 = a^3 + 3a^2b + 3b^2a + b^3$

You should notice that the coefficients of (the numbers before) a and b are:

1 1
1 2 1
1 3 3 1

If you continued expanding the brackets for higher powers, you would find that the sequence continues:

1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
etc

This sequence is known as **Pascal's triangle**. Each of the numbers is found by adding together the two numbers directly above it.
So the 20 in the last line is found by adding together 10 and 10. Each of the 10s in the line above are found by adding together a 6 and a 4.

So it is possible to expand (a + b) to any whole number power by knowing Pascal's triangle.

Example

Find $(3 + x)^3$

The power that we are expanding the bracket to is 3, so we look at the third line of Pascal's triangle, which is 1 3 3 1.
So the answer is: $3^3 + (3 \times 3^2 \times x) + (3 \times x \times 2^3) + x^3$ (we are replacing a by 3 and b by x in the expansion of $(a + b)^3$ above)

Generally

It is, of course, often impractical to write out Pascal's triangle every time, when all that we need to know are the entries on the nth line.

Clearly, the first number on the nth line is 1. The second number is n. The third number is:

$$\frac{n(n-1)}{1 \times 2}$$

In general, the rth number in the nth line is:

$$\frac{n!}{r!\,(n-r)!}$$ (which is ${}^nC_r$ on your calculator)

${}^nC_r$ is also often written as $\binom{n}{r}$ and is pronounced "n choose r".

The Binomial Theorem

The **Binomial Theorem** states that:

- $(a + b)^n = a^n + ({}^nC_1)a^{n-1}b + ({}^nC_2)a^{n-2}b^2 + \ldots + ({}^nC_{n-1})ab^{n-1} + b^n$

Example

Expand $(4 + 2x)^6$ in ascending powers of x up to the term in x^3

This means use the Binomial theorem to expand the terms in the brackets, but only go as high as x^3.
So to find the answer we substitute 4 for a in the Binomial theorem and 2x for 6:

$4^6 + ({}^6C_1)(4^5)(2x) + ({}^6C_2)(4^4)(2x)^2 + ({}^6C_3)(4^3)(2x)^3 + \ldots$

$= 4096 + (6 \times 1024 \times 2x) + (15 \times 256 \times 4x^2) + (20 \times 64 \times 8x^3) + \ldots$

$= 4096 + 12288x + 15360x^2 + 10240x^3 + \ldots$

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Algebraic Long Division

Algebraic long division is very similar to traditional long division (which you may have come across earlier in your education). The easiest way to explain it is to work through an example.

Example

Divide $2x^3 - 3x^2 - 3x + 2$ by $x - 2$

$$x - 2 \;\overline{)\; 2x^3 - 3x^2 - 3x + 2}$$

What do we have to multiply x by to get $2x^3$?
Answer: $2x^2$. We put this on the top line:

$$\begin{array}{r|l} & 2x^2 \\ \hline x - 2 & 2x^3 - 3x^2 - 3x + 2 \\ & 2x^3 - 4x^2 \end{array}$$

multiply the $2x^2$ by $(x - 2)$ and write the answer here

Subtract the $2x^3 - 4x^2$ from the numbers above & bring the next term down (the $-3x$)

$$\begin{array}{r|l} & 2x^2 \\ \hline x - 2 & 2x^3 - 3x^2 - 3x + 2 \\ & 2x^3 - 4x^2 \\ \hline & \quad\; x^2 - 3x \end{array}$$

Continue doing this over and over again.
What do you have to multiply x by to get x^2?

$$\begin{array}{r|l} & 2x^2 + x \\ \hline x - 2 & 2x^3 - 3x^2 - 3x + 2 \\ & 2x^3 - 4x^2 \\ \hline & \quad\; x^2 - 3x \\ & \quad\; x^2 - 2x \end{array}$$

$$\begin{array}{r|l} & 2x^2 + x \\ \hline x - 2 & 2x^3 - 3x^2 - 3x + 2 \\ & 2x^3 - 4x^2 \\ \hline & \quad\; x^2 - 3x \\ & \quad\; x^2 - 2x \\ \hline & \qquad\quad -x + 2 \end{array}$$

After each step, bring down the next term in the quotient. Continue until you have no terms left.

$$\begin{array}{r|l} & 2x^2 + x - 1 \\ \hline x - 2 & 2x^3 - 3x^2 - 3x + 2 \\ & 2x^3 - 4x^2 \\ \hline & \quad\; x^2 - 3x \\ & \quad\; x^2 - 2x \\ \hline & \qquad\quad -x + 2 \\ & \qquad\quad -x + 2 \\ \hline & \qquad\qquad\;\; 0 \end{array}$$

In this case, there is no remainder (hence the zero).

NB:
If the polynomial/ expression that you are dividing has a term in x missing, add such a term by placing a zero in front of it. For example, if you are dividing $x^3 + x - 4$ by something, rewrite it as $x^3 + 0x^2 + x - 4$.

For algebraic long division practise makes perfect- the best way to learn how to do them properly is to do loads of examples until you get them right every time!

The Remainder Theorem

When dividing one algebraic expression by another, more often than not there will be a remainder. It is often useful to know what this remainder is and it can often be calculated without going through the process of dividing. The rule is:

- If a polynomial f(x) is divided by ax - b, the remainder is f(b/a)

In the above example, $2x^3 - 3x^2 - 3x + 2$ was divided by x - 2.
Let $f(x) = 2x^3 - 3x^2 - 3x + 2$. In this case, a = 1, b = 2. The remainder is therefore f(2) $= 2\times2^3 - 3\times2^2 - 3\times2 + 2 = 0$, as we saw when we divided the whole thing out.

The Factor Theorem

This states:

- If (x – a) is a factor of the polynomial f(x), then f(a) = 0

In other words, if a polynomial f(x) can be divided by (x – a) without a remainder, then x = a is a root of f(x).

In the above worked example, f(2) = 0. This means that (x - 2) is a factor of the equation.

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Errors

Absolute Errors

Since no measurement is exact, there will always be the possibility of an error. Numbers are often rounded off to a certain number of significant figures or decimal places, and in such instances an absolute error can be calculated.
If the number 1200 is given to the nearest 100, for example, then the number can be anything between 1150 and 1250. It can therefore be written as 1200 ± 50. In this instance, the absolute error is 50.

- In general, if a number is a ± b, then the absolute error is b .

Relative Errors

The absolute error doesn't really tell us much about how big the error really is. For example, an absolute error of 1000 is very big if the number we are talking about is 3000, but it is small if we are talking about 100 000 000 000. The relative error incorporates the number that we are talking about.

- If a number is a ± b, then the relative error is b/a

Example

If 200 is correct to 2 significant figures, what is the relative error?
This can be written as 200 ± 5, since the highest number it could be is 205 and the lowest is 195. The relative error, therefore, is 5/200 = 1/40 = 0.025 .

Percentage Error

Percentage error = relative error × 100

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Functions

Introduction

The phrase 'y is a function of x' means that the value of y depends upon the value of x, so y can be written in terms of x (e.g. $y = 3x$).
If $f(x) = 3x$, and y is a function of x (i.e. $y = f(x)$), then the value of y when x is 4 is $f(4)$, which is found by replacing x's by 4's .

Example

If $f(x) = 3x + 4$, find $f(5)$ and $f(x + 1)$.

$f(5) = 3(5) + 4 = \underline{19}$
$f(x + 1) = 3(x + 1) + 4 = \underline{3x + 7}$

The inverse of a function

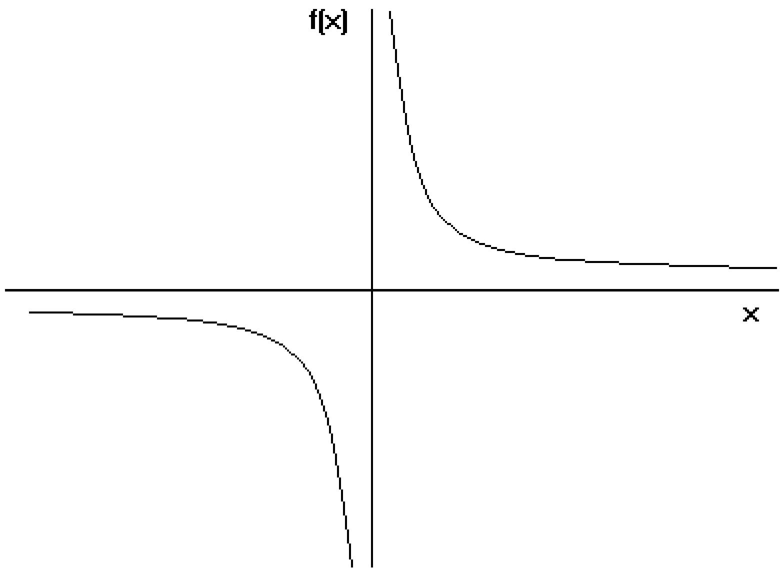
The inverse of a function is the function which reverses the effect of the original function. For example the inverse of $y = 2x$ is $y = \frac{1}{2}x$.
To find the inverse of a function, swap the x's and y's and make y the subject of the formula.

Example

Find the inverse of $f(x) = 2x + 1$
Let $y = f(x)$, therefore $y = 2x + 1$
swap the x's and y's:
$x = 2y + 1$
Make y the subject of the formula:
$2y = x - 1$, so $y = \frac{1}{2}(x - 1)$
Therefore $f^{-1}(x) = \underline{\frac{1}{2}(x - 1)}$
$f^{-1}(x)$ is the standard notation for the inverse of $f(x)$

Graphs

Functions can be graphed. A function is **continuous** if its graph has no breaks in it. An example of a discontinuous graph is $y = 1/x$, since the graph cannot be drawn without taking your pencil off the paper:



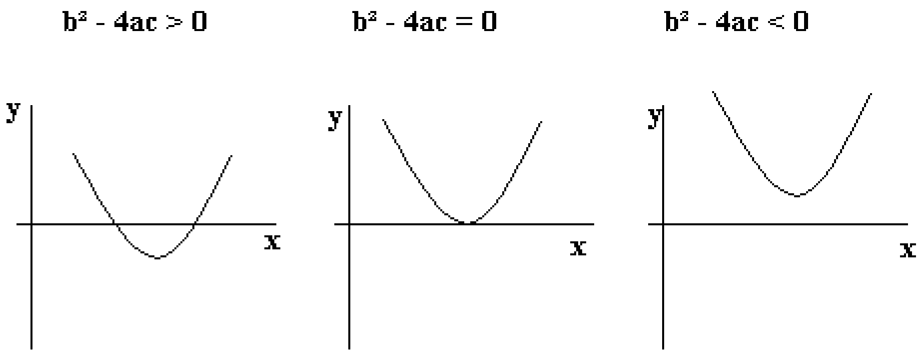
A function is **periodic** if its graph repeats itself at regular intervals, this interval being known as the period.
A function is **even** if it is unchanged when x is replaced by -x . The graph of such a function will be symmetrical in the y-axis. Even functions which are polynomials have even degrees (e.g. $y = x^2$).
A function is **odd** if the sign of the function is changed when x is replaced by -x . The graph of the function will have rotational symmetry about the origin (e.g. $y = x^3$).

Quadratic Functions

The quadratic formula is:

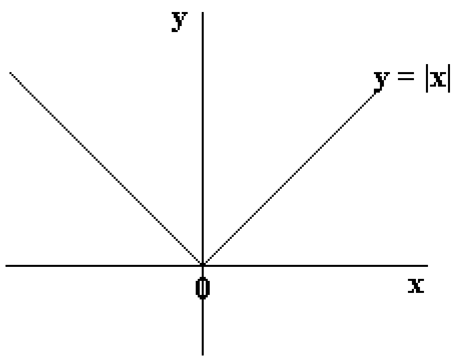
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

, where $y = ax^2 + bx + c$.
Since you only know how to take square roots of positive numbers, there are only real solutions if $b^2 - 4ac$ is greater or equal to 0. The expression $b^2 - 4ac$ is therefore important, and is known as the **discriminant**. If $b^2 - 4ac$ is less than zero, then there will be no values of x giving a value of y of zero, hence the graph of the curve will not cross the x-axis.



The Modulus Function

The modulus of a number is the magnitude of that number. For example, the modulus of -1 ($|-1|$) is 1. The modulus of x, $|x|$, is x for values of x which are positive and -x for values of x which are negative. So the graph of $y = |x|$ is $y = x$ for all positive values of x and $y = -x$ for all negative values of x:



Transforming graphs

If $y = f(x)$, the graph of $y = f(x) + c$ (where c is a constant) will be the graph of $y = f(x)$ shifted c units upwards (in the direction of the y-axis).
If $y = f(x)$, the graph of $y = f(x + c)$ will be the graph of $y = f(x)$ shifted c units to the left.
If $y = f(x)$, the graph of $y = f(x - c)$ will be the graph of $y = f(x)$ shifted c units to the right.
If $y = f(x)$, the graph of $y = af(x)$ is a stretch of the graph of $y = f(x)$, scale factor a, from the x-axis.

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Iteration

Iteration is a way of solving equations. You would usually use iteration when you cannot solve the equation any other way.
An iteration formula might look like the following:

$$x_{n+1} = 2 + \frac{1}{x_n}.$$

You are usually given a starting value, which is called x_0. If $x_0 = 3$, for example, you would substitute 3 into the original equation where it says x_n. This will give you x_1. (This is because if $n = 0$, $x_1 = 2 + 1/x_0$ and $x_0 = 3$).
$x_1 = 2 + 1/3 = 2.333\ 333$ (by substituting in 3).
To find x_2, substitute the value you found for x_1.
$x_2 = 2 + 1/(2.333\ 333) = 2.428\ 571$

Repeat this until you get an answer to a suitable degree of accuracy. This may be about the 5th value for an answer correct to 3s.f. In this example, $x_5 = 2.414...$

Example

a) Show that $x = 1 + \dfrac{11}{x - 3}$
is a rearrangement of the equation $x^2 - 4x - 8 = 0$.

b) Use the iterative formula $X_{n+1} = 1 + \dfrac{11}{x_n - 3}$
together with a starting value of $x_1 = -2$ to obtain a root of the equation $x^2 - 4x - 8 = 0$ accurate to one decimal place.

a) multiply everything by $(x - 3)$:
$x(x - 3) = 1(x - 3) + 11$
so $x^2 - 3x = x + 8$
so $x^2 - 4x - 8 = 0$

b) $x_1 = -2$
$x_2 = 1 + \dfrac{11}{-2 - 3}$ (substitute -2 into the iteration formula)
$= -1.2$
$x_3 = 1 + \dfrac{11}{-1.2 - 3}$ (substitute -1.2 into the above formula)

= -1.619

x_4 = -1.381

x_5 = -1.511

x_6 = -1.439

x_7 = -1.478

therefore, to one decimal place, <u>x = 1.5</u> .

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Logarithms

Logarithms

Logarithms are another way of writing indices.

- If $a = b^c$ then $c = \log_b a$

You may often see ln x and log x written, with no base indicated. It is generally recognised that this is shorthand:

- $\log_e x = \ln x$
- $\log_{10} x = \lg x$ or $\log x$ (on calculators)

Laws of logs

The properties of indices can be used to show that the following rules for logarithms hold:

- $\log_a x + \log_a y = \log_a (xy)$

- $\log_a x - \log_a y = \log_a (x/y)$

- $\log_a x^n = n\log_a x$

Example

Simplify: log 2 + 2log 3 - log 6
= log 2 + log 3^2 - log 6
= log 2 + log 9 - log 6
= log (2 × 9) - log 6
= log 18 - log 6
= log (18/6)
= log 3

NB: In the above example, I have not written what base each of the logarithms is to. This is because for the laws of logarithms, it doesn't matter what the base is, as long as all of the logs are to the same base.

Another important law of logs is as follows. This is a very useful way of changing the base (in this formula, the base *does* matter!). Most calculators can only work out ln x and $\log_{10} x$ (usually just written as 'log' on the button) so this formula can be very useful.

$$\log_a b = \frac{\log_c b}{\log_c a}$$

Example

Calculate, to 3s.f., $\log_3 5$

$$\log_3 5 = \frac{\log_{10} 5}{\log_{10} 3} = 1.46 \text{ (3s.f.)}$$

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Partial Fractions

It is possible to split many fractions into the sum or difference of two or more fractions. This has many uses (such as in integration).
At GCSE level, we saw how:

$$\frac{1}{(x + 1)} + \frac{4}{(x + 6)} = \frac{5(x + 2)}{(x + 1)(x + 6)}$$

The method of partial fractions allows us to split the right hand side of the above equation into the left hand side.

Linear Factors in Denominator

This method is used when the factors in the denominator of the fraction are linear (in other words do not have any square or cube terms etc).

Example

Split $\frac{5(x + 2)}{(x + 1)(x + 6)}$ into partial fractions.

We can write this as:

$$\frac{5(x + 2)}{(x + 1)(x + 6)} \equiv \frac{A}{(x + 1)} + \frac{B}{(x + 6)}$$

So now, all we have to do is find A and B.

$$\therefore \frac{5(x + 2)}{(x + 1)(x + 6)} \equiv \frac{A(x + 6) + B(x + 1)}{(x + 1)(x + 6)}$$ (putting the fractions over a common denominator)

$\therefore 5(x + 2) \equiv A(x + 6) + B(x + 1)$ (we have cancelled the denominators)

The above expression is an **identity** (hence ≡ rather than =). An identity is true for every value of x. This means that we can substitute any values of x into both sides of the expression to help us find A and B. When trying to work out these constants, try to choose values of x which will make the arithmetic easier. In this example, if we substitute x = -6 into the identity, the A(x + 6) term will disappear, making it much easier to solve.

when $x = -6$,

$5(-4) = B(-5)$

$\therefore B = 4$

when $x = -1$,

$5(1) = 5A$

$\therefore A = 1$

since $\dfrac{5(x + 2)}{(x + 1)(x + 6)} \equiv \dfrac{A}{(x + 1)} + \dfrac{B}{(x + 6)}$

the answer is $\dfrac{1}{(x + 1)} + \dfrac{4}{(x + 6)}$ (as we knew)

Cover Up Method

The "cover-up method" is a quick way of working out partial fractions, but it is important to realise that this only works when there are linear factors in the denominator, as there are here.

To put $\dfrac{5(x + 2)}{(x + 1)(x + 6)}$ into partial fractions using the cover up method:

cover up the $x + 6$ with your hand and substitute –6 into what's left, giving –5(-6 + 2)/(-6+1) = -20/-5 = 4. This tells you that one of the partial fractions is 4/(x + 6). Now cover up (x + 1) and substitute –1 into what's left to discover that the other partial fraction is 1/(x + 1) .

Repeated Factor in the Denominator

Remember, the above method is only for linear factors in the denominator. When there is a repeated factor in the denominator, such as $(x - 1)^2$ or $(x + 4)^3$, the following method is used.

Example

Split $\dfrac{x - 2}{(x + 1)(x - 1)^2}$ into partial fractions

This time we write:

$$\frac{x - 2}{(x + 1)(x - 1)^2} \equiv \frac{A}{(x + 1)} + \frac{B}{(x - 1)} + \frac{C}{(x - 1)^2}$$

Note that we have put a $(x - 1)$ **and** a $(x - 1)^2$ fraction in.
As before, all we do now is find the values of A, B and C, by putting them over a common denominator and then substituting in values for x.

$x - 2 \equiv A(x - 1)^2 + B(x - 1)(x + 1) + C(x + 1)$

let $x = 1$
$-1 = 2C$
$C = -\frac{1}{2}$

let $x = -1$
$-3 = 4A$
$A = -\frac{3}{4}$

let $x = 0$
$-2 = A - B + C$
$-2 = -\frac{3}{4} - B - \frac{1}{2}$
$B = \frac{3}{4}$

Therefore the answer is:

$$\frac{-3}{4(x+1)} + \frac{3}{4(x-1)} - \frac{1}{2(x-1)^2}$$

Quadratic Factor in the Denominator

This method is for when there is a square term in one of the factors of the denominator.

Example

$$\frac{2x - 1}{(x+1)(x^2+1)} \equiv \frac{A}{(x+1)} + \frac{Bx + C}{(x^2+1)}$$

Find A, B and C in the same way as above.

Note that it is **Bx + C** on the numerator of the fraction with the squared term in the denominator.

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Reduction to Linear Form

In order to show that data from a science experiment fits a rule, we have to be able to plot two variables on a graph so that a straight line relationship results (this will be the line of best fit if the data is experimental).

In questions, you may be given some data and you may be asked to plot y against 1/x . If the data lies in a straight line, you know, from the equation of a straight line ($y = mx + c$), that $y = m/x + c$, since 1/x is on the x-axis. You have therefore found the relationship between the data.

Example

V	5	10	15	20	25
R	149	175	219	280	359

By drawing a graph, show that these pairs of values may be regarded as approximations to values satisfying a relationship of the form $R = a + bV^2$.

In this question, you would plot R against V^2 . If a straight line comes out, then you know that (from $y = mx + c$) $R = mV^2 + c$ (which is just the same as $R = a + bV^2$, since m and c, a and b are just constants).
The second part of such a question may ask you to calculate the values of a and b. In this example, b is the gradient of the graph and a is the y-intercept (this can again be established by comparing the equation to $y = mx + c$), so you can read these values off from the graph.

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Sequences

nth Term

In the sequence 2, 4, 6, 8, 10... there is an obvious pattern. Such sequences can be expressed in terms of the nth term of the sequence. In this case, the nth term = 2n. To find the 1st term, put n = 1 into the formula, to find the 4th term, replace the n's by 4's: 4th term = 2 × 4 = 8.

Example

What is the nth term of the sequence 2, 5, 10, 17, 26... ?
The easiest way to find the nth term is by trial and error.

n =	1	2	3	4	5
n^2 =	1	4	9	16	25
$n^2 + 1$ =	2	5	10	17	26

This is the required sequence, so the nth term is $n^2 + 1$. For some sequences, there is no easy way of working out the nth term of a sequence, other than to try different possibilities.
Tips: if the sequence is going up in threes (e.g. 3, 6, 9, 12...), there will probably be a three in the formula, etc.
In many cases, square numbers will come up, so try squaring n, as above.
Also, the triangular numbers formula often comes up. This is $\frac{1}{2}n(n + 1)$.

Notation

The nth term of a sequence is sometimes written as U_n . So in the last example, $U_n = n^2 + 1$. The 5th term is therefore $U_5 = 25 + 1 = 26$.

Recurrence relation

This is where the next term of a sequence is defined using the previous term(s). For example, the recurrence relation for 2, 4, 8, 16, 32, ... would be: $U_1 = 2$, $U_n = 2(U_n\text{-}1)$. This tells us that the first term, U_1, is 2 and the next term of the sequence can be found by doubling the previous term.

Convergent Sequences

Sequences whose nth term approaches a finite number as n becomes larger are known as convergent sequences and the number to which the sequence converges is known as the limit of the sequence. For example: 10, 5, 2.5, 1.25, 0.625, ... converges (gets closer and closer) towards the limit zero.

See Also: Series (for arithmetic and geometric progressions)

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