

**PRIFYSGOL CYMRU, BANGOR**  
**UNIVERSITY OF WALES, BANGOR**



**YSGOLORIAETH MYNEDIAD**  
**ENTRANCE SCHOLARSHIPS**

**2003**

**MATHEMATEG**  
**MATHEMATICS**

## MATHEMATICS

Time allowed : 3 hours

*You may attempt any of the questions. Credit will be given for complete answers.***Question 1.** Express

$$\frac{1}{x(x^2 + 1)}$$

in partial fractions.

**Question 2.** Find the coordinates of the turning points and of the points of inflection of the curve

$$y = (2 - x)(1 + x^2)$$

Sketch the curve.

**Question 3.** Find the area of the region between the curves

$$y = x^2$$

and

$$y = 4x - x^2$$

**Question 4.** Simplify the expression  $\tan(x) + \tan^3(x)$  and hence calculate the integral

$$\int \tan(x) + \tan^3(x) dx$$

**Question 5.** Using a substitution evaluate the integral

$$\int_1^4 \frac{10\sqrt{x}}{(1 + x^{3/2})^2} dx$$

**Question 6.** Using the integration by parts method evaluate the integral

$$\int_1^2 \ln(x + x^2) dx$$

**Question 7.** Using the binomial theorem or otherwise find the coefficient of  $x^7$  in the expansion of

$$\left(2x^2 - \frac{1}{4x}\right)^5$$

**Question 8.** Find the centre and radius of the circle

$$x^2 + y^2 + 4x - 6y + 5 = 0$$

and hence find the length of the tangent to the circle from the point  $P(1, -1)$ .

**Question 9.** Calculate the derivative at  $x = 0$  of the function  $y$  given by

$$y = \frac{(x+1)(x-2)}{x-1}$$

**Question 10.** Calculate the gradient at  $(2, 3)$  to the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 2$$

**Question 11.** A stone is released from the top of a 100 metre tower at the same time as another stone is projected vertically upwards. The stones meet after 5 seconds. Find the speed of projection of the lower stone. Take  $g = 9.8 \text{ m s}^{-2}$ .

**Question 12.** A ship is sailing due east with velocity 20 mph. A submarine is 1 mile due south of the ship when it fires a torpedo which travels at 25 mph. In which direction must it be fired so as to hit the ship? How long will it take to reach the ship?

**Question 13.** A cricket ball is thrown from a height of  $4 \text{ ft}$  at an angle of 30 degrees to the horizontal and with a speed of  $80 \text{ ft/sec}^{-1}$ . It is caught by a fielder at a height of  $1 \text{ ft}$  from the ground. How far apart are the two players? Take  $g = 32 \text{ ft s}^{-2}$ .

**Question 14.** A uniform ladder stands on a rough surface of coefficient of friction  $\mu = 0.5$  and leans at 45 degrees to the vertical against a smooth wall. The ladder weighs 60 kg. How far up the ladder can a man weighing 200 kg climb before it slips?

**Question 15.** The rotation of the earth causes the weight of a body as measured on the equator by a spring balance to be diminished. Calculate this diminution when the true weight is  $100 \text{ kgm}$ , taking  $g = 9.8 \text{ m s}^{-2}$  and the radius of the earth as  $r = 6300 \text{ km}$ .

**Question 16.** The continuous random variable  $X$  has probability density function  $f$  defined by

$$f(x) = \lambda x(1-x) \quad 0 \leq x \leq 1.$$

Calculate the parameter  $\lambda$  and the mean and standard deviation of  $f$ .

**Question 17.** In an examination the marks are assumed to be normally distributed with mean  $\mu = 52.1$  and standard deviation  $\sigma = 4.9$ . A random sample of 100 scripts is analysed. What is the distribution for such a sample? What is the probability that the mean of such a sample will lie between 51 and 53?

**Question 18.** Telephone calls to an evening radio program occur randomly. On a typical evening the program receives 1000 calls. One evening the prime minister speaks on the national news. Design a significance test for the null hypothesis  $H_0$  at level  $\alpha = 0.05$  that there will be no effect on the number of calls.

**Question 19.** A sphere of radius  $a$  has a hole of diameter  $b$  drilled symmetrically through the middle. What is the depth of the hole?

**Question 20.** Five points are marked at random but strictly inside an equilateral triangle of size 2 metres. Explain why there must be two of the points distant less than one metre apart.

**Question 21.** A basketball player is known to score 85 % of the penalty shots he takes. Find the probability that out of 5 shots, the player will score at least 3.

**Question 22.** Mr Smith has to leave home early for an early office meeting. He does not want to wake up Mrs Smith at this hour so he does not turn the lights on. He goes to his sock drawer and chooses two of the socks. There are 5 black, 3 gray and 2 navy pairs of socks but they are all mixed up.

- (i) What is the probability that Mr Smith arrives at his office meeting with a matching pair of socks?
- (ii) To be on the safe side, Mr Smith decides to take a few more socks in his pocket so as to be sure that at least two of the socks he takes match. How many socks should he take altogether?