WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Level/Special Paper

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APPLIED MATHEMATICS S

A.M. FRIDAY, 2 July 1999

(3 Hours)

INSTRUCTIONS TO CANDIDATES

Answer **six** questions.

The only books of statistical tables that you may use in the examination are "Statistical Tables" by Murdoch and Barnes (Macmillan Press) **or** "Elementary Statistical Tables" (RND Publications).
Take $g = 9{\cdot}8\ \text{ms}^{-2}$.

INFORMATION FOR CANDIDATES

An electronic calculator will be required.

The booklet "Information for the use of candidates in Mathematics" is available and may be used.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A and B are two points on a vertical line, B being at a height h above A. A particle P is projected vertically upwards from A with initial speed $3\sqrt{(gh)}$, and at the same instant, a second particle Q is projected vertically upwards from B with initial speed $2\sqrt{(gh)}$.

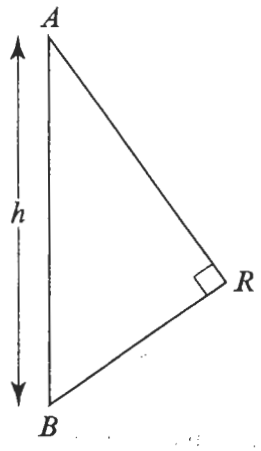
(a) Show that the particles collide at a height $\frac{5h}{2}$ above A. [4]

(b) Given that the particles are of equal mass and that the coefficient of restitution between them is $\frac{1}{4}$, show that

(i) Q's speed immediately after the collision is given by $\frac{13\sqrt{(gh)}}{8}$, and find a similar expression for P's speed at this time, [6]

(ii) Q returns to B at the same instant as P returns to A. [7]

2.



(a) The diagram shows a smooth ring R of mass m threaded on a light inextensible string whose ends are attached to two fixed points A and B. The point A is a distance h vertically above the point B. The ring is moving in a horizontal circle with constant angular speed ω about its centre that lies on AB. The section AR is inclined at an angle $\cos^{-1}(\frac{4}{5})$ to AB. The sections AR and BR are perpendicular to each other.

(i) Find the tension in the string. [3]

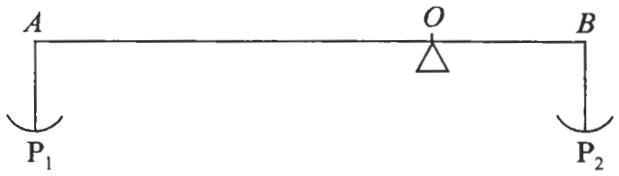
(ii) Show that $\omega^2 = \frac{175g}{12h}$. [4]

(b) The string is now replaced by two separate light inextensible strings AR and BR having lengths such that the ring moves in the same circle as in part (a) but with angular speed Ω. State the change that you have to make in your modelling assumptions and show that the motion is only possible when

$$\Omega^2 \geqslant \frac{25g}{16h}.$$ [8]

(c) Explain why, if the strings in (b) were replaced by thin rods of the same length, there would be no restriction on Ω. [2]

3.



The diagram shows a simple model of a balance with a thin light rod AB, free to turn about some point O of its length, carrying scale pans P_1 and P_2 suspended at A and B respectively. The point O is **not** at the centre of AB and the weight of the pans may **not** be neglected.

(*a*) A particular balance is balanced (i.e. in equilibrium with AB horizontal) with the scale pans empty. Using this balance, it was found that a weight W_1 N placed in P_1 was balanced by a weight of 42 N in P_2, and that a weight W_1 N placed in P_2 was balanced by a weight of 168 N in P_1. Find the value of W_1. [7]

(*b*) Another balance, which does not balance with the pans empty, is such that a weight of 80 N placed in P_1 is balanced by a weight of 50 N in P_2. Using this balance, it was found that a weight W_2 N placed in P_1 was balanced by a weight of 90 N in P_2 and that a weight W_2 N placed in P_2 was balanced by a weight of 180 N in P_1. Find the value of W_2. [10]

4. A particle P of mass m is free to move in the plane of a square $OABC$, of side $2a$, under the action of a resultant force mn^2 (**OP** + 2**PA** + **PB** + 2**PC**), where n is a constant. Unit vectors along OA and OC are denoted by **i** and **j** respectively. At time t, the position vector of P with respect to O is denoted by $x\mathbf{i} + y\mathbf{j}$.

(*a*) Find, in terms of x, y, a, m, n, **i** and **j**, an expression for the resultant force acting on P. [4]

(*b*) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = n^2(6a - 4x)$$

and obtain the general solution of this differential equation. [4]

(*c*) It is given that P was released from rest at the point Q on BC, where $CQ = \frac{3a}{2}$.

(i) Show that P can move only on the straight line through Q and perpendicular to BC. [4]

(ii) Obtain the differential equation satisfied by y. [1]

(iii) Show that the motion of P is simple harmonic and find the position vector of the centre, the period and amplitude of this motion. [4]

5. Gwyn and Peter play a series of frames of snooker against each other. The probability that Gwyn will win any particular frame is p and the probability that Peter will win any particular frame is q. Drawn frames are impossible in snooker. The result of any one frame is independent of the result of any other frame.

(*a*) One evening, Gwyn and Peter play a match in which the winner will be the first to win 4 frames.

(i) Find an expression in terms of p and q for the probability that Gwyn will win the match.

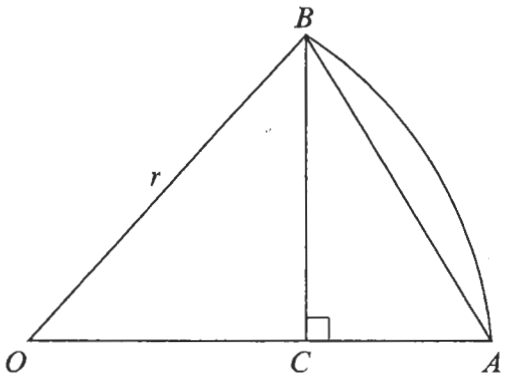
(ii) Calculate the probability that Gwyn will win the match for the particular case $p = 0{\cdot}65$. Comment on your answer. [8]

(*b*) The following evening, Gwyn and Peter play another match. This time they decide that the winner will be the first player to win at least 4 frames, while at the same time having a lead of at least 2 frames over his opponent. Thus if the score is 3 frames each, the match must continue until one of the players leads the other by two frames.

(i) Show that the probability that Gwyn will win the match by 5 frames to 3 is $20p^5q^3$.

(ii) Find and simplify an expression in terms of p and q for the probability that Gwyn will win the match. [9]

6.



The diagram shows the sector AOB of a circle having centre O and radius r, together with the chord AB. C is the foot of the perpendicular from B to OA. Angle AOB equals X radians, where X is a continuous random variable having probability density function f given by

$$f(x) = \sin x, \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$
$$f(x) = 0, \qquad \text{otherwise.}$$

(*a*) Find the expected value and variance of the area of **triangle** AOB. Deduce the expected value and variance of the length of BC. [12]

(*b*) Suppose that 200 such **sectors** are chosen at random. Find the probability that exactly 7 of these sectors will have an area of less than $\frac{r^2}{8}$. Give your answer correct to two decimal places. [5]

7. The random variable X has the cumulative distribution function given by

$$F(x) = 0 \qquad \text{for } x < 0,$$
$$F(x) = \left(\frac{x}{\theta}\right)^{\alpha} \qquad \text{for } 0 \leqslant x \leqslant \theta,$$
$$F(x) = 1 \qquad \text{for } x > \theta,$$

where θ and α are unknown positive constants.

(*a*) (i) Obtain an expression for μ, the mean of X, in terms of α and θ.

(ii) Show that the variance, σ^2, of X is given by

$$\sigma^2 = \frac{\alpha\theta^2}{(\alpha+2)(\alpha+1)^2}.$$ [6]

(*b*) In order to estimate α and θ, 100 observations were made on X and the following results calculated:

$$\Sigma x = 202{\cdot}3; \quad \Sigma x^2 = 459{\cdot}2.$$

(i) Estimate μ and σ^2.

(ii) By equating these estimates respectively to the expressions for μ and σ^2 found in (*a*), and solving the resulting simultaneous equations, find estimates for α and θ.

(iii) Given that $\alpha = 2$, estimate the standard error of your estimate of θ. [11]

8. The following random sample was taken from a N(μ, 1) distribution

1·83, 1·99, 0·23, 0·93, 1·70, −0·11, 2·02, −0·48, −0·54, 1·88

Each of three students is asked to suggest an appropriate statistic to test the hypotheses:

$$H_0 : \mu = 0 \text{ versus } H_1 : \mu > 0$$

(I) Ann suggests the number of positive observations in the sample.

(II) Brenda suggests the sum of all the observations in the sample.

(III) Chris suggests the maximum value in the sample.

(*a*) Find, for each student, the value of the chosen statistic and calculate its *p*-value with respect to the above hypotheses. [8]

(*b*) Calculate the least value of each test statistic that would lead to the acceptance of H_1 when a significance level of 5% is used. [9]